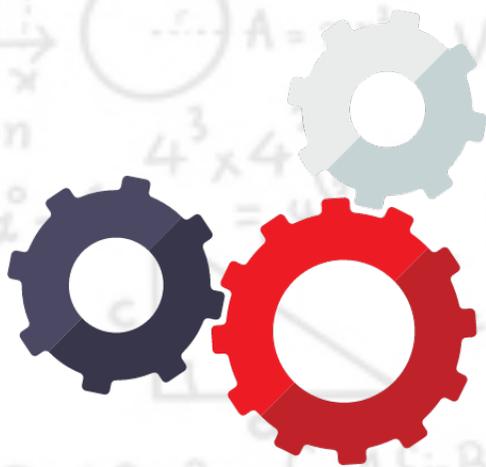


Purposeful Maths

# CALCULATION POLICY



Version 1  
January 2022

# Background, Purpose and Aims

Mathematics is a subject in which its learning episodes can be taught in multiple different ways, using multiple different methods; this is the case at many schools. This can cause significant confusion and cognitive overload for some students, especially lower attaining students.

The purpose of this document is to provide mathematics teachers, teachers of other subjects and staff who support students in mathematics lessons with an easy-reference guide to some the methods that could be employed in the teaching of mathematics.

In mathematics we aim to ensure thorough understanding of the subject content covered without relying on process memorisation. The methods outlined are one of the ways of accomplishing this.

This document will allow staff to synchronise their practise, to ensure students encounter the same methods throughout their mathematical journey, regardless of their teacher.

The aim is that this will provide consistency for students in the long-term and therefore aid in improving their outcomes.

We acknowledge that there may be other methods of teaching the topics included in this Policy, some of which may work better for other styles of teaching. The methods included are those that we have found beneficial for our students.

The Policy does not include all topics, nor does it include all approaches for all sub-topics. This is the first version of an ongoing project. Please do contact us with your feedback.



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@mrgreen\_maths

***Note: This is a working document and aims to take account of all research and best practise. This document does not aim for a 'one size fits all' approach, nor does it intend to stipulate how to teach, it is merely a mechanism to improve the consistency of students' mathematical experience.***

## Acknowledgements

*This document was written and created by Martin Green and Phil Bruce (Co-Creators of Purposeful Maths).  
We would also like to acknowledge the input and thoughts of the rest of our department during the implementation of this Policy.*

# Introduction

Throughout this document each approach is split into four stages: **Physical**, **Pictorial**, **Semi-Abstract** and **Abstract**. The idea is through a systematic approach students will begin, where possible, to explore mathematics by using physical manipulatives so that at the end of the process students should be able to form their own generalisations of mathematical rules.

Here are the four stages, along with explanations of each:

Stage	Name	Explanation
1	Physical	During the <b>Physical Stage</b> , students will have the opportunity to work with manipulatives and other physical objects in order to understand the mathematical concept. There will be times where this is not possible, in these cases students should begin at Pictorial Stage.
2	Pictorial	During the <b>Pictorial Stage</b> , students should be able to pictorially or diagrammatically represent ideas discovered during the Physical Stage.
3	Semi-Abstract	During the <b>Semi-Abstract Stage</b> , students should begin to be able to form generalisations and therefore not be fully reliant on a diagram. During this stage, however, a student's understanding should still partly be reliant or aided by a diagram.
4	Abstract	During the <b>Abstract Stage</b> , students should no longer require a diagram to understand the concept. Students should have formed <u>comprehensive</u> generalisations during which the underlying mathematics is <u>fully understood</u> .

Ideally, by the end of Year 11, all students will fully understand all of the abstract methods taught, along with the underlying mathematics. Realistically, however, this will not be the case for all students. By the end of Year 11, some students may not grasp the abstract nature of some topics, however this approach allows students to succeed using multiple representation and thus have a greater chance of success.

For example, when expanding single brackets, some lower attaining students may not fully understand why when asked to expand the answer is because the terms inside the brackets need to be multiplied by the term outside. However students may see where this comes from when this is represented using a Semi-Abstract Grid Method.

**Curriculum Area:**  
**NUMBER**

**Addition and Subtraction**

**Factors**

**Directed Number**

**Fractions**

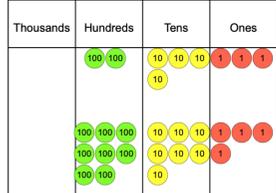
**Percentages**

## COLUMN ADDITION (INTEGERS AND DECIMALS)

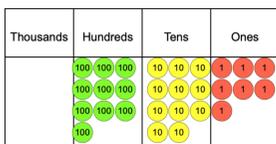
Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
Students should be able to use <b>place value counters</b> to represent the two numbers and understand the idea of carrying.	Students should draw the manipulatives as a diagram in their books, clearly showing where the values are being carried.	Students should use the ideas in the Physical and Pictorial stages and partition the numbers.	Students should use the traditional column addition method, carrying where appropriate
<b>Example(s)</b>	<b>Example(s) - These should be seen in books</b>		

Calculate  $243 + 874$

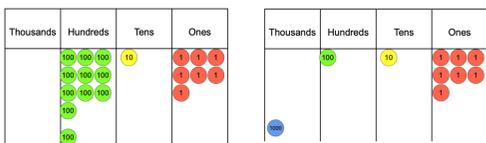
Students should represent the numbers in a place value table using place value counters.



Students should then physically move the counters together starting from the leftmost column.



Students should then understand that 10 1's make 10, 10 10's make 100, etc. and that they can replace these 10 counters with 1 of the next highest counters.

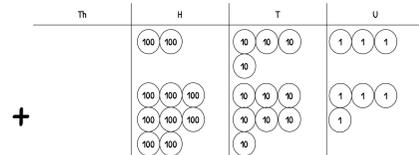


(Notice the added counter is placed lower than the existing ones, this will aid with the idea of carrying later)

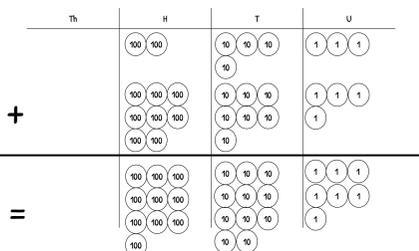
Students can then just count the leftover counters.

$$243 + 874 = 1117$$

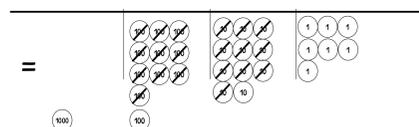
Students should draw the place value table and counters in their books.



Students should then total up each place value column and draw the correct number of each counter.



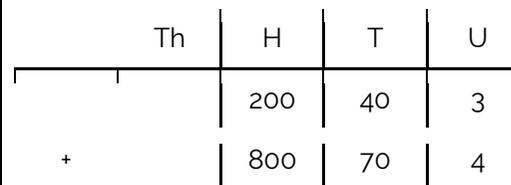
Where there are more than 10 of a place value counter, students should cross out those which would total the next value up and draw an additional counter in the appropriate column, this should be below those counters that are already there.



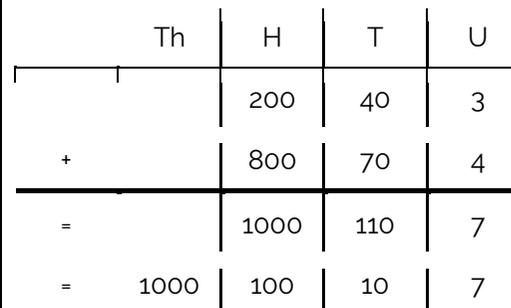
Students can then just count the non-crossed out circles.

$$243 + 874 = 1117$$

Students should partition the numbers and represent these values in a place value table



Students should the sum each column, carrying where appropriate.



Students should then sum the final row.

$$243 + 874 = 1117$$

Students should be able to use a formal column method of addition

$$\begin{array}{r}
 243 \\
 + 874 \\
 \hline
 1117 \\
 1 \quad 1
 \end{array}$$

# FACTORS

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
Students should use a manipulative (likely double sided counters or multi-link cubes) and maneuver them to understand factors and multiples	Students use the ideas formed in the physical stage to use the squares in exercise books to draw rectangles	Students should use a diagram to find the factors of the required number	Students should be able to list the factors immediately
<b>Example(s)</b>	<b>Example(s) - These should be seen in books</b>		

## Find all of the factors of 12

Students should represent the number using the manipulative



Students should then understand that amount of counters along and the amount of counters up are factors of the value.

Students should then work **systematically**, adding one to the height and attempting to form a rectangle.

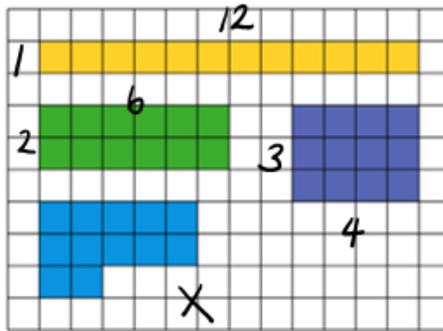
Testing if 1 is a factor: [From this, students should form the understanding that 1 is a factor of every number]			Testing 12:  1 and 12 are factors of 12
Testing if 2 is a factor:  Since this forms a perfect rectangle <b>2 and 6 are factors of 12</b>	Testing if 3 is a factor:  Since this forms a perfect rectangle <b>3 and 4 are factors of 12</b>	Testing if 4 is a factor:  Since this forms a perfect rectangle <b>4 and 3 are factors of 12</b>	
Testing if 4 is a factor:  Since this forms a perfect rectangle <b>4 and 3 are factors of 12</b>	Testing if 5 is a factor:  Since this DOES NOT form a perfect rectangle <b>5 is NOT a factor of 12.</b>	Testing if 6 is a factor:  <b>6 and 2 are factors of 12</b>	

Students should identify that the factors come in pairs and that they only need to be listed once.

Factors of 12 are 1, 2, 3, 4, 6, 12

Students should use the squares in their books to sketch the value in rectangles.

From the Physical stage, students should understand that the factors come in pairs and therefore only need to be drawn once.

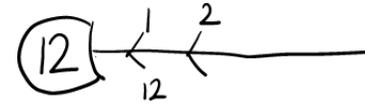


Students should clearly explain why certain values are NOT factors.

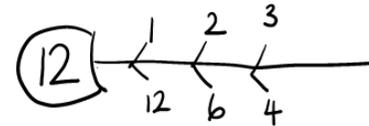
E.g. the light blue diagram above does not form a perfect rectangle, therefore 5 is NOT a factor of 12.

Factors of 12 are 1, 2, 3, 4, 6, 12

Students should work systematically, starting at 1 and increasing by one each time and determining if it is a factor and the corresponding value.



Students should understand that as soon as they reach a number that is already in the diagram they have found all of the factors.



Students MUST then list the factors.

Factors of 12 are 1, 2, 3, 4, 6, 12

Students will not need any form of diagram at this stage.

Students should understand the meaning of factors and will likely be able to find factors in their head.

Factors of 12 are 1, 2, 3, 4, 6, 12

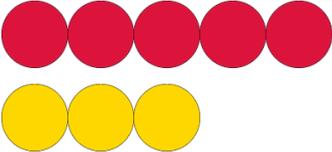
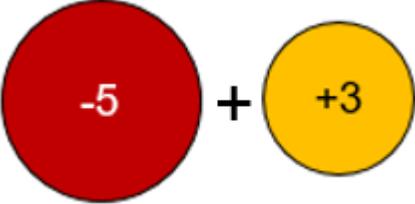
## DIRECTED NUMBER

### GENERAL TEACHING & LEARNING POINTS

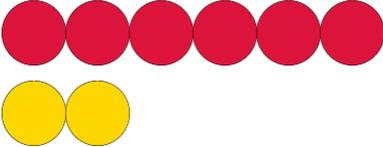
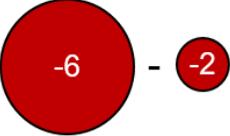
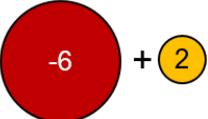
The following apply to all four operations with negative numbers:

- Teachers should not use the term "minus", instead use the words "negative" and "subtract"
- "Two negatives make a positive" should NEVER be used – this is not mathematically accurate
- The idea of zero-pairs should be emphasised throughout as this will also be used in an algebra context
- The idea that subtraction is the **additive inverse** should be shown, e.g. subtracting 3 is the same as adding -3
- Students should be confident with using double-sided counters to represent numbers prior to attempting four operations

### Addition

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
<p>Students should be able to use <b>double-sided counters</b> to represent the two numbers and the idea of zero pairs.</p>	<p>Students should draw the manipulatives as a diagram in their books, clearly showing the zero-pairs.</p>	<p>Students should use the ideas in the Physical and Pictorial stages to be able to determine, through using 'scaled' double-sided counter diagrams to determine if the answer is positive or negative.</p>	<p>Students should be able to state an answer without the use of a diagram.</p>
<b>Example(s)</b>	<b>Example(s) - These should be seen in books</b>		
<p>Calculate <math>-5 + 3</math> "Negative 5 add 3"</p>			
<p>Students should use double-sided counters to represent the -5 using five -1 counters and then physically add three +1 counters.</p> <div style="text-align: center;">  </div> <p>Students should then identify the zero-pairs in their representation and remove them.</p> <div style="text-align: center;">  </div> <p>Students should then be able to identify their answer from their remaining counters.</p> <p style="text-align: center;"><math>-5 + 3 = -2</math></p>	<p>Students should represent the calculation as in the Physical Stage, however, for short it may be beneficial for students to use - and + signs instead of drawing full circles.</p> <div style="text-align: center;"> <p>-   -   -   -   -</p> <p>+   +   +</p> </div> <p>Students should then identify the zero-pairs in their representation <b>and circle them</b>. They could show they have been removed by crossing through them.</p> <div style="text-align: center;">  </div> <p>Students should then be able to identify their answer from their remaining symbols.</p> <p style="text-align: center;"><math>-5 + 3 = -2</math></p>	<p>Students should represent this by drawing a larger negative circle plus a smaller positive circle. <a href="https://mathsbot.com/manipulatives/directedCounters">https://mathsbot.com/manipulatives/directedCounters</a> may be useful for this representation.</p> <p>For weaker students a number line representation in addition may also be beneficial.</p> <div style="text-align: center;">  </div> <p>Students should be able to determine from the calculation that there will be more negative counters than positive ones in this calculation, <b>therefore the answer must be negative.</b></p> <p style="text-align: center;"><math>-5 + 3 = -2</math></p>	<p>Students should be able to perform the calculation without any working.</p> <p style="text-align: center;"><math>-5 + 3 = -2</math></p>

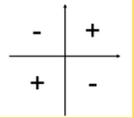
## Subtraction

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
Students should be able to use <b>double-sided counters</b> to represent the two numbers and the idea of zero pairs.	Students should draw the manipulatives as a diagram in their books, clearly showing the zero-pairs.	Students should use the ideas in the Physical and Pictorial stages be able to determine, through using 'scaled' double-sided counter diagrams to determine if the answer is positive or negative.	Students should be able to state an answer without the use of a diagram.
<b>Example(s)</b>	<b>Example(s) - These should be seen in books</b>		
<p>Calculate <math>-6 - -2</math>                      "Negative 6 subtract -2"</p>			
<p>Students should use double-sided counters to represent the -6.  <a href="https://mathsbot.com/manipulatives/doubleSidedCounters">https://mathsbot.com/manipulatives/doubleSidedCounters</a> may be useful for this representation.</p> <p>Students should then understand that subtracting -2 is equivalent to +2.</p> <p>Students should then add these counters to their representation.</p>  <p>Students should then identify the zero-pairs in their representation and remove them.</p>  <p>Students should then be able to identify their answer from their remaining counters.</p> $-6 - -2 = -4$	<p>Students should represent the calculation as in the Physical Stage, however, for short it may be beneficial for students to use - and + signs instead of drawing full circles.</p> $\begin{array}{cccccc} - & - & - & - & - & - \\ + & + & & & & \end{array}$ <p>Students should then identify the zero-pairs in their representation <b>and circle them</b>. They could show they have been removed by crossing through them</p>  <p>Students should then be able to identify their answer from their remaining symbols.</p> $-6 - -2 = -4$	<p>Students should represent this by drawing a larger negative circle subtract a smaller negative circle.  <a href="https://mathsbot.com/manipulatives/directedCounters">https://mathsbot.com/manipulatives/directedCounters</a> may be useful for this representation.</p> <p>For weaker students a number line representation in addition may also be beneficial.</p>  <p>Students should then apply the additive inverse as they have done in previous stages and re-draw.</p>  <p>Notice the size of the second circle does not change.</p> <p>Students should be able to determine from the calculation that there are will be more negative counters than positive ones in this calculation, <b>therefore the answer must be negative.</b></p> $-6 - -2 = -4$	<p>Students should be able to perform the calculation without any working.</p> $-6 - -2 = -4$

# Multiplication

## GENERAL TEACHING & LEARNING POINTS

- Students should be confident using factors and multiples and the use of a co-ordinate grid.
- Students should understand that the opposite side of the x or y axis involves multiplying by -1 (flipping the counters over)



### Stage 1 Physical

Students should be able to use **double-sided counters** and a numbered grid to represent the problem as an area model.

### Example(s)

### Stage 2 Pictorial

Students should draw the manipulatives as a diagram in their books.

### Stage 3 Semi-Abstract

Students should use the ideas in the Physical and Pictorial stages to be able to determine an answer, through using the area of a rectangle model.

### Stage 4 Abstract

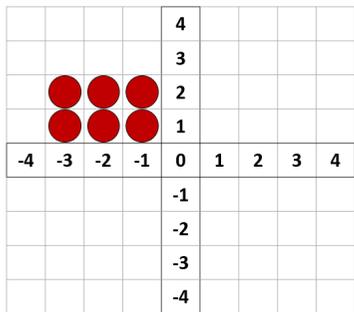
Students should be able to state an answer without the use of a diagram.

### Example(s) - These should be seen in books

Calculate  $-3 \times 2$   
"Negative 3 multiplied by 2"

For consistency students could represent the first number on the x-axis and the second the y-axis.

Students should use the grid as shown below:

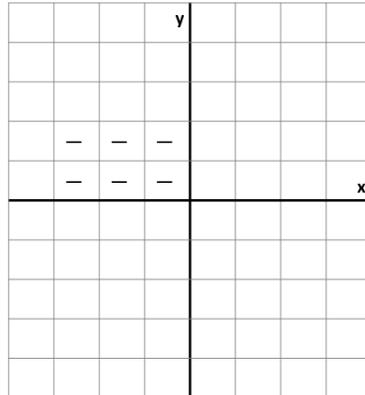


The counters are -3 along the x axis and +2 lots of this (so +2 on the y-axis).

Students should then be able to identify their answer from the counters on their grid

$$-3 \times 2 = -6$$

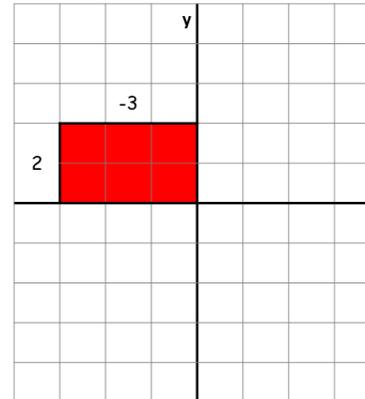
Students should draw a small grid in their exercise books and place either + or - symbols in each square to represent the same idea as in the physical stage.



Students should then be able to identify their answer from the signs in their grid

$$-3 \times 2 = -6$$

Students should draw a similar diagram to those in the Physical and Pictorial stage, however at this stage students should represent the calculation through an area model.



Students should then apply their knowledge of understanding

$$-3 \times 2 = -6$$

Students should be able to perform the calculation without any working.

$$-3 \times 2 = -6$$

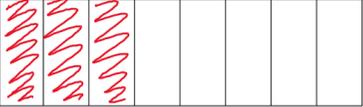
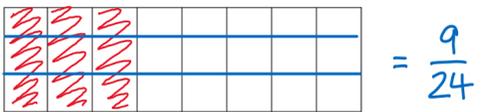
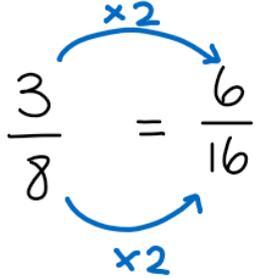
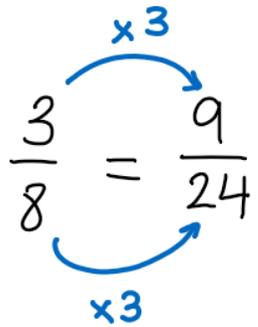
# Division

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
<p>Students should be able to use <b>double-sided counters</b> and a numbered grid to represent the problem as an area model.</p>	<p>Students should draw the manipulatives as a diagram in their books.</p>	<p>Students should use the ideas in the Physical and Pictorial stages be able to determine an answer, through using the area of a rectangle model.</p>	<p>Students should be able to state an answer without the use of a diagram.</p>
<b>Example(s)</b>	<b>Example(s) - These should be seen in books</b>		
<p>Calculate <math>-8 \div -2</math> "Negative 8 divided by negative 2"</p>			
<p>Students will be confident with the + - grid below.</p> <div style="text-align: center;"> </div> <p>Students should know that for this calculation -8 represents the counters required, and this would be represented in quadrant 2 or 4.</p> <p>Student should understand that one dimension needs to be -2.</p> <p>Either representation below could be used.</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> </div> <div style="text-align: center;"> </div> </div> <p>Students should then be able to identify the solution to the problem is the length/width of the other dimension of the diagram.</p> <p style="text-align: center;"><math>-8 \div -2 = 4</math></p>	<p>Students will be confident with the + - grid below.</p> <div style="text-align: center;"> </div> <p>Students should know that for this calculation -8 represents the counters required, and this would be represented in quadrant 2 or 4.</p> <p>Student should understand that one dimension needs to be -2.</p> <p>Either representation below could be used.</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> </div> <div style="text-align: center;"> </div> </div> <p>Students should then be able to identify the solution to the problem is the length/width of the other dimension of the diagram.</p> <p style="text-align: center;"><math>-8 \div -2 = 4</math></p>	<p>Students should represent the calculation as an area as with Dividing Numbers.</p> <p>Either representation below could be used.</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> </div> <div style="text-align: center;"> </div> </div> <p>Students should then be able to identify the solution to the problem is the length/width of the other dimension of the diagram.</p> <p style="text-align: center;"><math>-8 \div -2 = 4</math></p>	<p>At this stage students should be able to perform the calculation without the need for any working.</p> <p style="text-align: center; font-size: 1.2em;"><math>-8 \div -2 = 4</math></p>

# EQUIVALENT FRACTIONS

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
N/A	Students should then be able to represent each fraction as a diagram, one split horizontally and the other vertically	Students should use a scale factor approach to find equivalent fractions.	Students should be able to state equivalent fractions.
<b>Example(s)</b>	<b>Example(s) - These should be seen in books</b>		

Write down three fractions that are equivalent to  $\frac{3}{8}$

<p><b>This may be added at a later date.</b></p>	<p>Students should represent the <math>\frac{3}{8}</math> fraction using a diagram, split either horizontally OR vertically.</p>  <p>Students should then understand that as long as this is <b>equally</b> split horizontally (or vertically if the original diagram is split horizontally) then the fractions will be equivalent to <math>\frac{3}{8}</math></p>  <p style="text-align: right;"><math>= \frac{6}{16}</math></p>  <p style="text-align: right;"><math>= \frac{9}{24}</math></p>	<p>Students should confidently understand that if the numerators and denominators are multiplied or divided by the same factor then the two fractions will be equivalent.</p> <p>This multiplication should be shown at this stage.</p>  	<p>At this stage students should be able to immediately state fractions that are equivalent to <math>\frac{3}{8}</math></p>
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## CONVERTING BETWEEN MIXED NUMBERS AND IMPROPER FRACTIONS

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract																																																								
N/A	Students should represent the initial quantity using a bar diagram.	Students at this stage should be able to determine the number of parts in the wholes plus any extras.	Students should be able to state the equivalent improper fraction/mixed number without any working.																																																								
<b>Example(s)</b>	<b>Example(s) - These should be seen in books</b>																																																										
Write $\frac{11}{5}$ as a mixed number.																																																											
<p><b>This may be added at a later date.</b></p>	<p>Students should understand that, since the denominator is 5, each whole one is split into 5 pieces.</p> <p>Students should draw a diagram to show the 11 parts.</p> <div style="display: flex; align-items: center; justify-content: center;"> <table border="1" style="border-collapse: collapse; text-align: center; margin-right: 10px;"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr> <tr><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>11</td><td></td><td></td><td></td><td></td></tr> </table> <div style="margin-left: 10px;"> <p>  whole</p> <p>  whole</p> <p><math>\frac{1}{5}</math></p> <p><math>= 2\frac{1}{5}</math></p> </div> </div> <p>Students should then identify the whole parts and the fraction parts and combine.</p>	1	2	3	4	5	6	7	8	9	10	11					<p>Students should be encouraged to think of how many <b>whole</b> 5s are in 11, and what the remainder is.</p> $\frac{11}{5} = 2 \text{ remainder } 1$ $= 2\frac{1}{5}$	<p>At this stage students should be able to immediately state the answer.</p> $\frac{11}{5} = 2\frac{1}{5}$																																									
1	2	3	4	5																																																							
6	7	8	9	10																																																							
11																																																											
Write $3\frac{2}{7}$ as a mixed number.																																																											
<p><b>This may be added at a later date.</b></p>	<p>Students should understand that, since the denominator is 7, each whole one is split into 7 pieces.</p> <p>Students should then count the number of parts.</p> <div style="display: flex; align-items: center; justify-content: center;"> <div style="margin-right: 10px;"> <p>3 {</p> <p>2/7 {</p> </div> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr> <tr><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td></tr> <tr><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td><td>21</td></tr> <tr><td>22</td><td>23</td><td></td><td></td><td></td><td></td><td></td></tr> </table> <div style="margin-left: 10px;"> <p><math>\frac{23}{7}</math></p> </div> </div>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23						<p>Students should <b>partly</b> rely on the diagram at this stage.</p> <p>Students should be able to determine the number of sevenths in the three whole ones and add the extras.</p> <div style="display: flex; align-items: center; justify-content: center;"> <div style="margin-right: 10px;"> <p>3 {</p> <p>2/7 {</p> </div> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr> <tr><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td></tr> <tr><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td><td>21</td></tr> <tr><td>22</td><td>23</td><td></td><td></td><td></td><td></td><td></td></tr> </table> <div style="margin-left: 10px;"> <p><math>3 \times 7 = 21</math></p> <p><math>21 + 2 = 23</math></p> <p><math>= \frac{23}{7}</math></p> </div> </div>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23						$3\frac{2}{7} = \frac{23}{7}$
1	2	3	4	5	6	7																																																					
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22	23																																																										

## ADDING AND SUBTRACTING FRACTIONS

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
N/A	Students should then be able to represent each fraction as a diagram, one split horizontally and the other vertically	Students should be able to determine the LCM of the two denominators.	Students should be able to add and subtract fractions without the need for a diagram; understanding the need for the LCM.
<b>Example(s)</b>	<b>Example(s) - These should be seen in books</b>		

Calculate  $\frac{2}{5} + \frac{1}{3}$

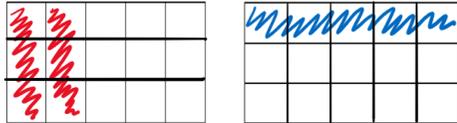
**This may be added at a later date.**

Students should represent each of the two fractions as separate diagrams.

ONE DIAGRAM SHOULD BE SPLIT VERTICALLY AND THE OTHER HORIZONTALLY.

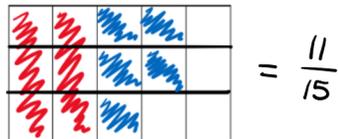


Students should then identify the LCM in order to find a common denominator and split each representation into this number of parts - See Equivalent Fractions.



Students should then understand that now all the small parts in the diagram are the same size, these can be combined.

For addition, students should add the parts in both diagrams to a single diagram.

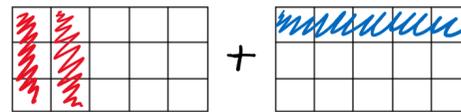


[For subtraction, students should cross through a part in the first diagram for each part in the second diagram]



Students should be able to determine the LCM of the two denominators, 15 in this case.

Students should then draw diagrams to represent these.



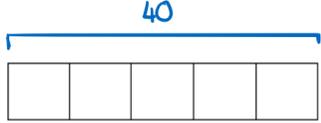
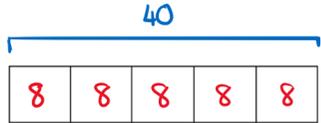
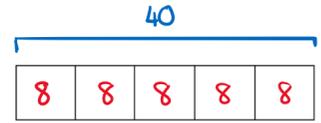
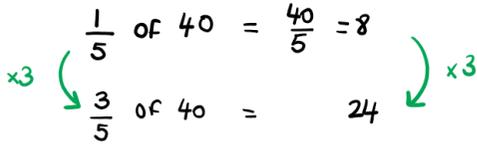
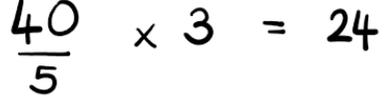
$$\frac{2}{5} = \frac{6}{15} \quad \frac{1}{3} = \frac{5}{15}$$

$$\frac{6}{15} + \frac{5}{15}$$

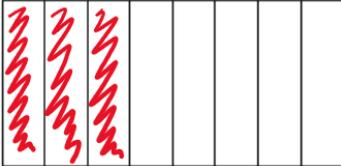
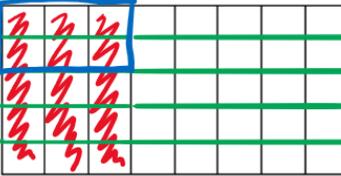
$$= \frac{11}{15}$$

$$\frac{2}{5} + \frac{1}{3} = \frac{6}{15} + \frac{5}{15} = \frac{11}{15}$$

## FRACTIONS OF AMOUNTS

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
N/A	Students should be able to represent the problem using a bar model.	Students should use a proportion idea first identifying a unit fraction and then scaling up.	Students should use a simple written method that shows finding a unit fraction and then multiplying.
<b>Example(s)</b>	<b>Example(s) - These should be seen in books</b>		
Find $\frac{3}{5}$ of 40			
<p><b>This may be added at a later date.</b></p>	<p>Students should represent the problem as a bar model, splitting the whole bar into the number of parts shown in the denominator and clearly showing the total.</p> <div style="text-align: center;">  </div> <p>Students should then work out the value of one part, students should then add this to their diagram.</p> <div style="text-align: center;">  </div> <p style="text-align: center;"><math>\frac{1}{5}</math> of 40 = <math>\frac{40}{5} = 8</math></p> <p>Students should then clearly show how they have obtained the <math>\frac{3}{5}</math></p> <div style="text-align: center;">  </div> <p style="text-align: center;"><math>\frac{1}{5}</math> of 40 = <math>\frac{40}{5} = 8</math></p> <p style="text-align: center;"><math>\frac{3}{5}</math> of 40 = <math>8 \times 3 = 24</math></p>	<p>Students should find <math>\frac{1}{5}</math> and then multiply this by 3.</p> <div style="text-align: center;">  </div>	<p>Students should show find <math>\frac{1}{5}</math> and then multiply this by 3 as a single calculation.</p> <div style="text-align: center;">  </div>

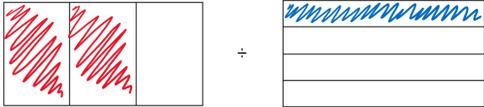
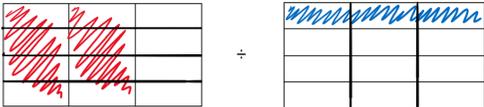
# MULTIPLYING FRACTIONS

GENERAL TEACHING & LEARNING POINTS	<ul style="list-style-type: none"> <li>To be consistent with diagrams, students should understand that multiplying fractions means the second a fraction OF the first, e.g. <math>\frac{1}{6} \times \frac{1}{2}</math> means <math>\frac{1}{2}</math> of <math>\frac{1}{6}</math></li> </ul>		
Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
N/A	Students should be able to represent the problem using a bar model.	Students should clearly show their written method, simplifying their answer.	Students at this stage should look for common factors to cancel to avoid larger numerators and denominators.
Example(s)	Example(s) - These should be seen in books		
Calculate $\frac{3}{8} \times \frac{2}{5}$			
<p>This may be added at a later date.</p>	<p>Students should draw a diagram to represent the <math>\frac{3}{8}</math></p>  <p>Students should then find <math>\frac{2}{5}</math> of the shaded part. Students should divide this into 5 part.</p>  <p>Students should circle the <math>\frac{2}{5}</math></p> $= \frac{6}{40}$ $= \frac{3}{20}$	<p>Students should complete the calculation without the need for a diagram. It is important that students include the <math>\frac{3 \times 2}{8 \times 5}</math> step to clearly show their understanding.</p> $\frac{3}{8} \times \frac{2}{5}$ $= \frac{3 \times 2}{8 \times 5}$ $= \frac{6}{40}$ $= \frac{3}{20}$	<p>Students should be encouraged to spot possible calculations after the first step, as shown below.</p> $\frac{3}{8} \times \frac{2}{5}$ $= \frac{3 \times \cancel{2}^1}{\cancel{4}^1 \times 5}$ $= \frac{3 \times 1}{4 \times 5}$ $= \frac{3}{20}$

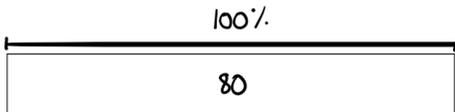
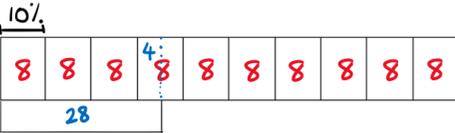
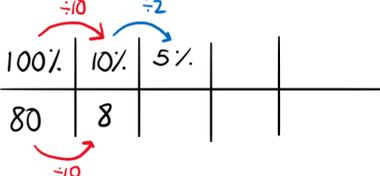
## DIVIDING FRACTIONS

GENERAL TEACHING & LEARNING POINTS	<ul style="list-style-type: none"> <li>"Keep Flip Change" should never be used</li> <li>Students should be encouraged to "multiply by the reciprocal"</li> <li>Students should be encouraged to think of <math>\frac{2}{3} \div \frac{1}{4}</math> as "How many <math>\frac{1}{4}</math> are there in <math>\frac{2}{3}</math>"</li> </ul>		
Stage 1 <b>Physical</b>	Stage 2 <b>Pictorial</b>	Stage 3 <b>Semi-Abstract</b>	Stage 4 <b>Abstract</b>
N/A	Students should be able to represent the problem using a bar model.	Students should clearly show their written method, simplifying their answer.	Students at this stage should clearly show division by a fraction is equivalent to multiplying by its reciprocal.
<b>Example(s)</b>	<b>Example(s) - These should be seen in books</b>		

Calculate  $\frac{2}{3} \div \frac{1}{4}$

<p><b>This may be added at a later date.</b></p>	<p>Students should represent each fraction in the calculation as separate diagrams, splitting one vertically and the other horizontally.</p>  <p>Students should use their knowledge of Equivalent Fractions to split their diagrams so that the parts in each diagram are equal sizes.</p>  <p>Students should understand that students need to determine</p> <p>how many  in </p> <p>Students should then circle the groups of three parts</p>  <p style="text-align: center;">2 Whole <math>\frac{2}{3}</math> left</p> <p style="text-align: center;"><math>= 2\frac{2}{3}</math></p>	<p>Students should understand that if they multiply both fractions by the reciprocal of the second, this reduces the calculation to a multiplication, and division by 1.</p> $\frac{2}{3} \div \frac{1}{4}$ $\xrightarrow{\times \frac{4}{1}} \left(\frac{2}{3} \times \frac{4}{1}\right) \div \left(\frac{1}{4} \times \frac{4}{1}\right) \xrightarrow{\times \frac{4}{1}}$ $= \frac{8}{3} \div 1$ $= \frac{8}{3}$ $= 2\frac{2}{3}$	<p>Students should represent this multiplication by the reciprocal as a single calculation.</p> $\frac{2}{3} \div \frac{1}{4}$ $= \frac{2}{3} \times \frac{4}{1}$ $= \frac{8}{3}$ $= 2\frac{2}{3}$
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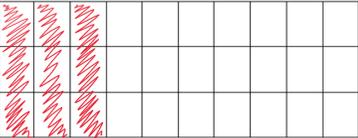
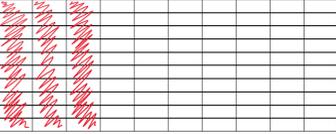
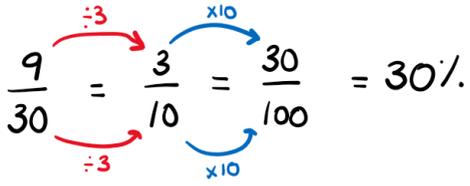
## PERCENTAGE OF AMOUNTS

GENERAL TEACHING & LEARNING POINTS																							
Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract																				
N/A	Students should be able to represent the problem using a bar model.	Students should use a written method, <b>set out using a proportion table</b> .	Students should use a written, <b>multiplier</b> , method in order to calculate the required percentage.																				
Example(s)	Example(s) - These should be seen in books																						
Calculate 35% of 80																							
<p><b>This may be added at a later date.</b></p>	<p>Students should use a bar model to represent the initial 100%.</p> <div style="text-align: center;">  </div> <p>Students should then split their bar model into a suitable number of parts.</p> <div style="text-align: center;">  </div> <p style="text-align: center;"><math>\frac{80}{10} = 8</math></p> <p>Students should then determine the correct percentage by using a second bar underneath</p> <div style="text-align: center;">  </div> <p style="text-align: center;"><math>\frac{80}{10} = 8</math></p>	<p>Students should represent the initial amount at 100% in a proportion table.</p> <div style="text-align: center;"> <table border="1" style="margin: auto;"> <tr> <td style="padding: 5px;">100%</td> <td style="width: 20px;"></td> <td style="width: 20px;"></td> <td style="width: 20px;"></td> <td style="width: 20px;"></td> </tr> <tr> <td style="padding: 5px;">80</td> <td></td> <td></td> <td></td> <td></td> </tr> </table> </div> <p>Students should then calculate a suitable percentage(s) to use to calculate the final amount, (10% and 5% in this case).</p> <div style="text-align: center;">  </div> <p>Students should complete their proportion table to obtain the required final percentage.</p> <p>Students should clearly state their final answer.</p> <div style="text-align: center;"> <table border="1" style="margin: auto;"> <tr> <td style="padding: 5px;">100%</td> <td style="padding: 5px;">10%</td> <td style="padding: 5px;">5%</td> <td style="padding: 5px;">30%</td> <td style="padding: 5px;">35%</td> </tr> <tr> <td style="padding: 5px;">80</td> <td style="padding: 5px;">8</td> <td style="padding: 5px;">4</td> <td style="padding: 5px;">24</td> <td style="padding: 5px;">28</td> </tr> </table> <p style="margin-top: 10px;"><math>35\% \text{ of } 80 = 28</math></p> </div>	100%					80					100%	10%	5%	30%	35%	80	8	4	24	28	<p>Students should understand that 'of' means multiply and determine the multiplier.</p> <div style="text-align: center; margin-top: 20px;"> <math display="block">0.35 \times 80 = 28</math> </div>
100%																							
80																							
100%	10%	5%	30%	35%																			
80	8	4	24	28																			

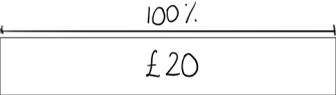
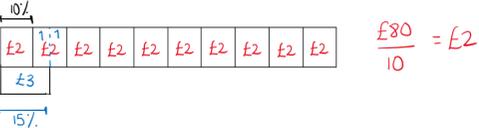
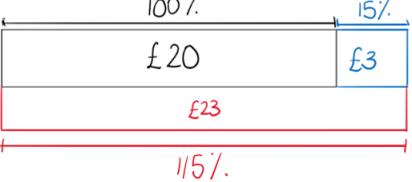
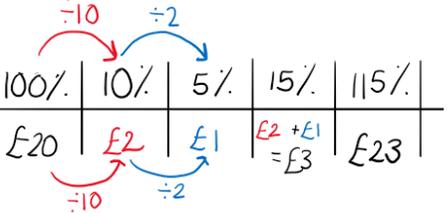
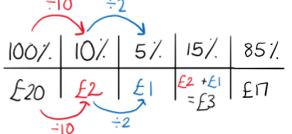
## ONE QUANTITY AS A PERCENTAGE OF ANOTHER

GENERAL TEACHING & LEARNING POINTS			
Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
N/A	Students should be able to represent the problem using a bar model.	Students should use a written method using equivalent fractions.	Students should understand that they can write their calculation as a single line of working, which could be entered into a calculator if permitted.
Example(s)	Example(s) - These should be seen in books		

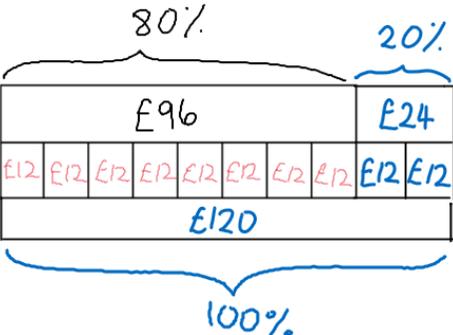
Zuzanna scored 9 out of 30 on a maths test. Work out her score as a percentage.

<p><b>This may be added at a later date.</b></p>	<p>Students should represent the problem as a diagram.</p> <p>They should divide the total amount into an appropriate amount, usually a factor of 100.</p> <div style="text-align: center;">  <math>\frac{9}{30}</math> </div> <p>Students should understand that percentage means 'per 100'. They should then split their diagram into 100 parts. It may be appropriate to split their diagram using two stages.</p> <div style="text-align: center;">  <math>\frac{3}{10}</math> </div> <p style="text-align: center;">Followed by</p> <div style="text-align: center;">  <math>\frac{30}{100} = 30\%</math> </div>	<p>Students should use their understanding obtained in the Pictorial Stage and the idea that this relates back to equivalent fractions, to show their working as below:</p> <div style="text-align: center;">  <math display="block">\frac{9}{30} = \frac{3}{10} = \frac{30}{100} = 30\%</math> </div>	$\frac{9}{30} \times 100 = 30\%$
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## PERCENTAGE INCREASE/DECREASE

GENERAL TEACHING & LEARNING POINTS	<ul style="list-style-type: none"> <li>Students should understand that increasing (or decreasing) by a percentage is equivalent to finding a percentage of an amount above (or below) 100%.</li> </ul>		
Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
N/A	Students should be able to represent the problem using a bar model.	Students should use a proportion table to find the required percentage.	Students should be able to use a calculator, through a multiplier, to determine the final amount.
Example(s)	Example(s) - These should be seen in books		
Increase £20 by 15%			
<p><b>This may be added at a later date.</b></p>	<p>Students should use a bar model to represent the initial 100%.</p> <div style="text-align: center;">  <p>A horizontal bar representing 100% with the value £20 written inside.</p> </div> <p>Students should then use the method shown in Finding Percentage of an Amount, to find the percentage to increase (or decrease) by.</p> <div style="text-align: center;">  <p>A bar divided into 10 equal segments, each labeled £2. A bracket above the first segment is labeled 10%. To the right, the calculation <math>\frac{£20}{10} = £2</math> is shown.</p> </div> <p>Students should then add on the £3 to the end of their diagram, clearly labelling this as 115%.</p> <div style="text-align: center;">  <p>A bar model where the first 10 segments are labeled 100% and £20, and the next 3 segments are labeled 15% and £3. A red bracket below the entire bar is labeled 115% and £23.</p> </div>	<p>Students should use their understanding of proportionality to set their working out in a proportion table, as shown below.</p> <div style="text-align: center;">  <p>A proportion table with columns for 100%, 10%, 5%, 15%, and 115%. The first row contains the percentages. The second row contains the corresponding values: £20, £2, £1, £2 + £1 = £3, and £23. Red arrows show £20 ÷ 10 = £2 and £2 × 2 = £4. Blue arrows show £20 ÷ 2 = £10 and £10 ÷ 2 = £5.</p> </div>	<p>Students should understand that 'of' means multiply and determine the multiplier.</p> <div style="text-align: center; font-size: 1.2em;"> <math>1.15 \times 20 = £23</math> </div>
	<p style="text-align: center;">Decrease £20 by 15%</p> <div style="text-align: center;">  <p>A bar model where the first 9 segments are labeled 95% and £17, and the last 1 segment is labeled 15% and £3. A red bracket below the first 9 segments is labeled 95%.</p> </div>	<p style="text-align: center;">Decrease £20 by 15%</p> <div style="text-align: center;">  <p>A proportion table with columns for 100%, 10%, 5%, 15%, and 85%. The first row contains the percentages. The second row contains the corresponding values: £20, £2, £1, £2 + £1 = £3, and £17. Red arrows show £20 ÷ 10 = £2 and £2 × 2 = £4. Blue arrows show £20 ÷ 2 = £10 and £10 ÷ 2 = £5.</p> </div>	<p style="text-align: center;">Decrease £20 by 15%</p> <div style="text-align: center; font-size: 1.2em;"> <math>0.85 \times 20 = £17</math> </div>

## REVERSE PERCENTAGES

GENERAL TEACHING & LEARNING POINTS	<ul style="list-style-type: none"> <li>Students should understand that increasing (or decreasing) by a percentage is equivalent to finding a percentage of an amount above (or below) 100%.</li> </ul>		
Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
N/A	Students should be able to represent the problem using a bar model.	Students should use a proportion table to find the required percentage.	Students should be able to use a calculator, through a multiplier, to determine the final amount.
Example(s)	<b>Example(s) - These should be seen in books</b>		
A shop has a 20% off sale. A pair of trainers have a sale price of £96. How much were the trainers before the sale?			
<p><b>This may be added at a later date.</b></p>	<p>Students should use a bar model to represent the discounted amount.</p> <div style="text-align: center;"> <math>80\%</math>   </div> <p>Students should then use the method shown in Finding Percentage of an Amount, to find the value of 100%.</p> <div style="text-align: center;"> <math>80\%</math>   </div> <div style="text-align: center;"> <math>80\%</math>      <math>20\%</math>   </div>	<p>Students should use their understanding of proportionality to set their working out in a proportion table, as shown below.</p> <div style="text-align: center;"> <math>\begin{array}{c c c} \div 8 &amp; \times 10 &amp; \\ \hline 80\% &amp; 10\% &amp; 100\% \\ \hline \pounds 96 &amp; \pounds 12 &amp; \pounds 120 \end{array}</math> <p style="text-align: center;">Original Price = £120</p> </div>	$80\% \text{ of Original Amount} = \pounds 96$ $0.8 \times \text{original Amount} = \pounds 96$ $\text{original Amount} = \frac{\pounds 96}{0.8}$ $= \pounds 120$ <p>(The first line of working could be omitted here)</p>

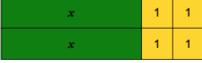
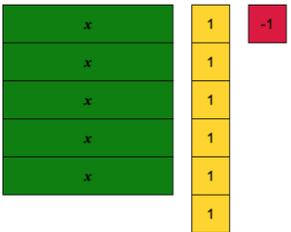
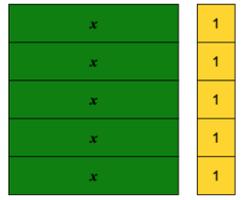
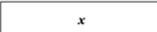
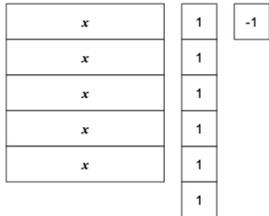
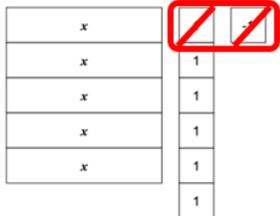
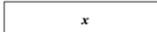
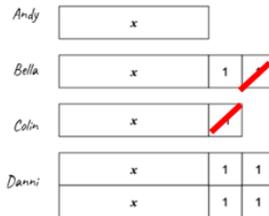
**Curriculum Area:**  
**ALGEBRA**

**Forming Expressions**  
**Substitution**  
**Simplifying Expressions**  
**Solving Linear Equations**  
**Linear Sequences**  
**Laws of Indices**  
**Expanding Single Brackets**  
**Factorising Single Brackets**  
**Linear Inequalities**  
**Linear and Quadratic Graphs**

# FORMING EXPRESSIONS

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
Students should be able to represent a problem using manipulative (cuisenaire rods or algebra tiles).	Students should draw the manipulatives as a diagram in their books, clearly collecting like terms and identifying the zero-pairs.	Students should use a diagram and identify the zero-pairs and cross them out.	Students should use algebra only to be able to collect like terms to find the total.
<b>Example(s)</b>	<b>Example(s) - These should be seen in books</b>		

Andy is  $x$  years' old. Bella is two years' older than Andy. Colin is one year younger than Andy. Danni is twice as old as Bella.  
Write an expression for the **total** age of all four people.

<p>Represent each person's age using cuisenaire rods, place each person above each other to show comparisons.</p> <div style="margin-bottom: 10px;"> <p>Andy </p> <p>Bella </p> <p>Colin </p> <p>Danni </p> </div> <p>Students should collect like terms (see Collecting Like Terms section)</p> <div style="margin-bottom: 10px;">  </div> <p>Students should then use the idea of <b>zero-pairs</b> to simplify</p> <div style="margin-bottom: 10px;">  </div> <p>Students should understand that this represents</p>	<p>Represent each person's age using cuisenaire rods, place each person above each other to show comparisons.</p> <div style="margin-bottom: 10px;"> <p>Andy </p> <p>Bella </p> <p>Colin </p> <p>Danni </p> </div> <p>Students should collect like terms (see Collecting Like Terms section)</p> <div style="margin-bottom: 10px;">  </div> <p>Students should then use the idea of <b>zero-pairs</b> to simplify. Students should circle, label and cross out the zero-pairs</p> <div style="margin-bottom: 10px;">  </div> <p>Students should then be able to count the 's and the 1's and that this represents</p>	<p>Represent each person's age using cuisenaire rods, place each person above each other to show comparisons.</p> <div style="margin-bottom: 10px;"> <p>Andy </p> <p>Bella </p> <p>Colin </p> <p>Danni </p> </div> <p>Students should then use the idea of <b>zero-pairs</b> to simplify, without the need to show the like terms together. Students should circle, label and cross out the zero-pairs.</p> <div style="margin-bottom: 10px;">  </div> <p>Students should then be able to count the remaining 's and 1's and that this represents</p>	<p>Students should represent each person's age as an algebraic expression.</p> <p><b>A</b> <math>x</math></p> <p><b>B</b> <math>x+2</math></p> <p><b>C</b> <math>x-1</math></p> <p><b>D</b> <math>2x+4</math> <i>This could be represented as <math>2(x+2)</math> depending on student's ability</i></p> <p>Students should then represent this as one expression clearly showing each expression either added or subtracted.</p> <p style="text-align: center;"><math>x + (x+2) + (x-1) + (2x+4)</math></p> <p>Notice that each expression is in brackets to ensure that students do not make sign errors with negatives.</p>
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## SUBSTITUTION INTO ALGEBRAIC EXPRESSIONS

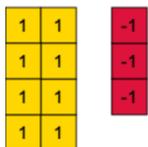
Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
Students should be able to algebra tiles to represent the expression in the problem, then replace the variable tiles with numerical tiles.	Students should draw the manipulatives as a diagram in their books, clearly replacing the variable with 1 or -1 tiles.	Students now be able to use the initial diagram and replace the variables with a value.	Students should now be able to apply their understanding of algebraic notation to substitution. At this stage, students will not need a diagram.
<b>Example(s)</b>	<b>Example(s) - These should be seen in books</b>		

If  $x = 3$  find the value of the expression  $4x - 3$

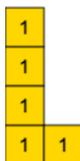
Using Algebra Tiles to represent the expression



Students should then replace  $x$  tiles with 3 1 tiles.



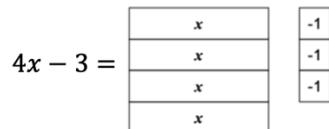
Students should then use the idea of **zero-pairs** to simplify



Students can then count the tiles to determine the answer.

5

Students should draw the algebra tiles representation in their books

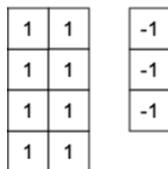


Students should clearly state (through a diagram) the value of each variable:

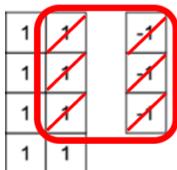


Students should then redraw their diagrams replacing the  $x$  bars with 3 1 tiles.

Where possible, students should be encouraged to make the number of 1 tiles the same size as the  $x$  tile it is replacing.



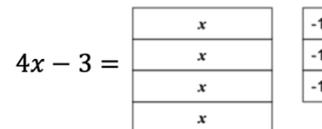
Students should then use the idea of **zero-pairs** to simplify. Students should circle, label and cross out the zero-pairs.



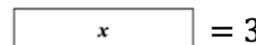
Students can then count the tiles to determine the answer.

5

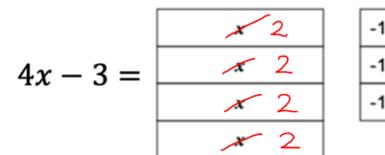
Students should draw the algebra tiles representation in their books



Students should clearly state (through a diagram) the value of each variable, using a number rather than 1s and -1s:



Students should then replace the  $x$  tiles by crossing out the  $x$  in their diagram and replacing it with the value of  $x$ .



Students should then calculate the value of the expression:

$$2 + 2 + 2 + 2 - 3$$

$$= 4 \times 2 - 3$$

$$= 5$$

Students should be able to re-write the algebraic expression with clear algebraic understanding

$$4x - 3$$

$$= 4 \times x - 3$$

$$= 4 \times 2 - 3$$

$$= 8 - 3$$

$$= 5$$

Higher attaining students should be discouraged from using the multiplication sign as above and instead should be encouraged to use brackets:

$$4x - 3$$

$$= 4(2) - 3$$

$$= 8 - 3$$

$$= 5$$



# SOLVING LINEAR EQUATIONS

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
Students should be able to represent a problem using manipulative algebra tiles.	Students should draw the manipulatives as a diagram in their books.	Students should be able to relate the diagrams of the algebra tiles to the equations.	Students should be able to use a formal balancing method to solve the equation.
<b>Example(s)</b>	<b>Example(s) - These should be seen in books</b>		

Solve the equation  $3x - 1 = 5$

Students should use Algebra Tiles to represent the equation.

Students should lay tiles on a **mini-whiteboard** so that students can write down their process. Students should then use the idea of zero-pairs to eliminate ones on one side.

Students should note down on each side of the equation what process they have done to keep the equation balanced.

Students should then remove any zero-pairs and arrange the ones evenly against the number of x's as shown below

Students should then determine how to find the value of a single x. write this process below any previous operations, and circle this

Students should draw the algebra tiles in their books.

Students should draw the x tiles first and then work in columns to add the ones, as shown below:

Students could use highlighters to make drawing the tiles easier and quicker.

Students should then use zero-pairs in order to eliminate the 1 or -1 tiles from one side of the equation.

Students should note down what they have added to both sides of the equation in order to do this.

Students should then cross out any zero-pairs.

If students have lined the x and 1s in columns systematically then they will be able to easily identify the value of 1 x tile.

Students should clearly write down that they have divided/multiplied below the last operation.

Some students may find it useful to circle the groups which give the value of one x.

Students should represent the equation in the same way as in the pictorial stage, but with the addition of a formal balancing method also shown.

Students should then form zero-pairs as in the pictorial stage. Students should also include what they have done in the balancing method.

Students should then use a similar diagram to the one used in the pictorial stage.

The student's final solution should be clearly written underneath any diagram or working.

At this stage students should be comfortable with solving equations without the need for a diagram and can solely rely on a formal balancing method.

Students however, at least initially, should show the operation that they have used in each stage of working.

# LINEAR SEQUENCES

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
Students should be able to represent a problem using manipulative, e.g. Cuisenaire Rods.	Students should draw the manipulatives as a diagram in their books.	Students should be able to identify the term-to-term rule and comparing this to the relevant times table.	Students should be able to determine the nth term rule by mentally comparing the sequence to the relevant times table.
<b>Example(s)</b>	<b>Example(s) - These should be seen in books</b>		

Work out the nth term rule for the sequence 4, 7, 10, 13, ...

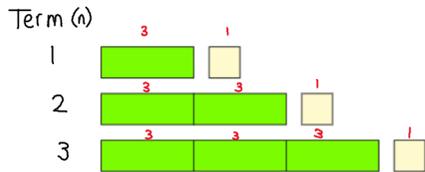
It will be beneficial if students use the manipulatives on top of a whiteboard for this stage.

Students should identify that the sequence increases by 3 each time.

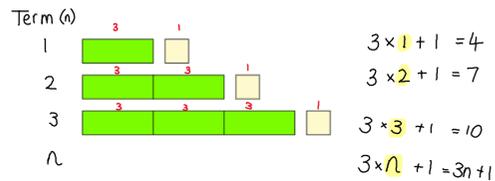


Students should then know that the sequence is linked to the 3 times table.

Students should then represent this using the manipulative. Since the sequence increases by 3, the 3 block will be needed.



Notice the term is also included and the value of each block is written above.

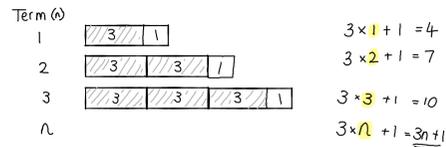


Students should then write each term as a multiple of the term-to-term rule, as shown on the right above.

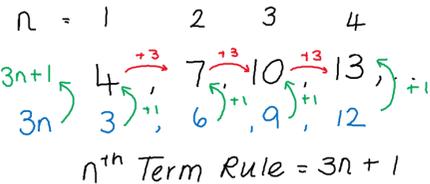
Students should identify that the sequence increases by 3 each time.



Students should then know that the sequence is linked to the 3 times table.



It may be useful for weaker students to shade in the term-to-term blocks to make them easier to count and distinguish.



Students should first write in the term numbers above the sequence.

Students should then determine the term-to-term rule, shown in **red** above.

Students should then know that the sequence is linked to the 3 times table and therefore the sequence  $3n$  is needed. This should be written below, shown in **blue** above.

Students should then determine how they obtain the sequence from the  $3n$  row, this is shown in **green** above. This can be done either as shown above or by subtracting (this may be useful for quadratics).

Students may be able to determine the nth term rule immediately at this stage and state the answer of  $3n+1$ .

Students however **MUST** be able to clearly justify why by explaining that the sequence is "one more than the three times table, therefore the nth term rule is  $3n+1$ ."

# LAWS OF INDICES

GENERAL TEACHING & LEARNING POINTS

The following apply to the multiplication, division and power on power rule:

- Teachers should ensure thorough understanding through writing out the meaning of each term as repeated multiplication at first instance.
- When confident through using this repeated multiplication, only then should students be shown the rule

## Multiplication Law

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
N/A	N/A	Students should understand the idea of an index and write out the meaning of each term.	Students should now be able to apply their understanding of like terms without the need to write out any terms.
Example(s)		Example(s) - These should be seen in books	

Write  $4a \times 3a^2$  as a single power of  $a$ .

NOT  
APPLICABLE

It may be useful to use colour to differentiate between the terms in the question.

Students at this stage should clearly write out what each term means as a repeated multiplication.

$$\begin{aligned} & \underline{4a} \times \underline{3a^2} \\ & = 4 \times a \times 3 \times a \times a \end{aligned}$$

Students should then understand that multiplication is **associative** and therefore the order of multiplication is irrelevant.

$$\begin{aligned} & \underline{4a} \times \underline{3a^2} \\ & = 4 \times a \times 3 \times a \times a \\ & = 4 \times 3 \times a \times a \times a \end{aligned}$$

Students should then evaluate

$$\begin{aligned} & \underline{4a} \times \underline{3a^2} \\ & = 4 \times a \times 3 \times a \times a \\ & = 4 \times 3 \times a \times a \times a \\ & = 12 \times a^3 \end{aligned}$$

Then leading to a final answer, **without a multiplication sign**

$$\begin{aligned} & \underline{4a} \times \underline{3a^2} \\ & = 4 \times a \times 3 \times a \times a \\ & = 4 \times 3 \times a \times a \times a \\ & = 12 \times a^3 \\ & = 12a^3 \end{aligned}$$

Only when confident with the previous layout, students should be able to write their working as:

$$\begin{aligned} & 4a \times 3a^2 \\ & = 4 \times 3 \times a^{1+2} \\ & = 12a^3 \end{aligned}$$

Students should be able to state the final answer to the question without the need for any working.

$$\begin{aligned} & 4a \times 3a^2 \\ & = 12a^3 \end{aligned}$$

## Division Law

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
N/A	N/A	Students should understand the idea of an index and write out the meaning of each term.	Students should now be able to apply their understanding of like terms without the need to write out any terms.
<b>Example(s)</b>		<b>Example(s) - These should be seen in books</b>	
Write $12b^5 \div 2b^3$ as a single power of b.			
NOT APPLICABLE	<p style="text-align: center;">Students should be encouraged to write the original problem as a fraction, and then split this fraction into two parts</p> $12b^5 \div 2b^3$ $= \frac{12 \times b^5}{2 \times b^3}$ $= \frac{12}{2} \times \frac{b^5}{b^3}$ <p style="text-align: center;">Students at this stage should then write the full meaning of each index</p> $= \frac{12}{2} \times \frac{b \times b \times b \times b \times b}{b \times b \times b}$ <p style="text-align: center;">Students should then simplify the fractions by cancelling/</p> $= \frac{\overset{6}{\cancel{12}}}{\underset{1}{\cancel{2}}} \times \frac{b \times b \times \cancel{b} \times \cancel{b} \times b}{\cancel{b} \times \cancel{b} \times b}$ <p style="text-align: center;">Students should then state their final answer</p> $= \frac{6}{1} \times b^2$ $= 6b^2$	<p style="text-align: center;">Students should begin the second part of this stage in the same way as previous.</p> $12b^5 \div 2b^3$ $= \frac{12 \times b^5}{2 \times b^3}$ $= \frac{12}{2} \times \frac{b^5}{b^3}$ <p style="text-align: center;">Students should then apply the law of index explicitly.</p> $= 6 \times b^{5-3}$ $= 6b^2$	<p style="text-align: center;">Students should be able to state the final answer to the question without the need for any working.</p> $12b^5 \div 2b^3$ $= 6b^2$

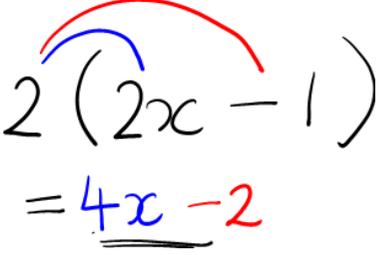
## Power-on-Power Law

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
N/A	N/A	Students should understand the idea of an index and write out the meaning of each term.	Students should now be able to apply their understanding of like terms without the need to write out any terms.
<b>Example(s)</b>		<b>Example(s) - These should be seen in books</b>	
Write $(2b^4)^3$ as a single power of b.			
<b>NOT APPLICABLE</b>	$(4x^2)^3$ $= \underline{4x^2} \times \underline{4x^2} \times \underline{4x^2}$ $= \underline{4} \times \underline{4} \times \underline{4} \times \underline{x \times x} \times \underline{x \times x} \times \underline{x \times x}$ $= 64 \times x^6$ $= 64x^6$	$(4x^2)^3$ $= 4^3 \times (x^2)^3$ $= 64 \times x^{2 \times 3}$ $= 64x^6$	<p style="text-align: center;">Students should be able to state the final answer to the question without the need for any working.</p> $(4x^2)^3$ $= 64x^6$

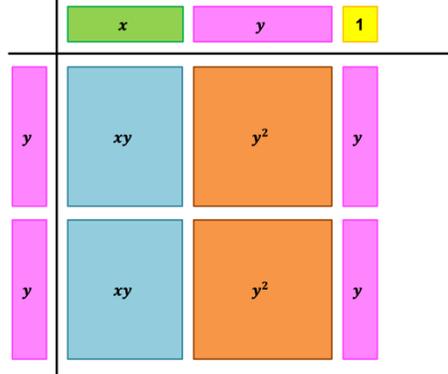
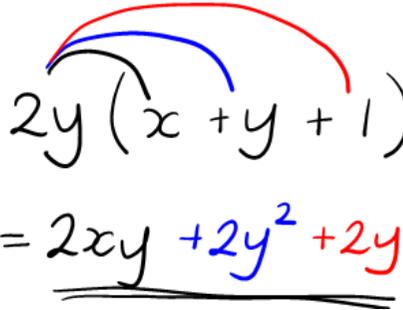
## EXPANDING SINGLE BRACKETS

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
Students should be able to use <b>algebra tiles</b> to represent an area model.	Students should be able to link expanding brackets to the areas of rectangles. This should be represented in an adapted grid method that illustrates different variables as different sizes.	Students should extend their knowledge in the Concrete stage to remove the idea of lengths whilst still associating the working to area if required	Students should not need to use a grid. students at this stage should understand that each term inside the bracket must be multiplied by the term outside.
<b>Example(s)</b>	<b>Example(s) - These should be seen in books</b>		

Expand  $2(2x - 1)$

 <p style="font-size: small;">Students should understand that means "two lots of". Students should then see that there are 4 'x's and 2 '1's, so</p>	<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="width: 10%;">x</td> <td style="width: 40%;">2x</td> <td style="width: 10%;">-1</td> </tr> <tr> <td style="width: 10%;">2</td> <td style="width: 40%;">Area = 4x</td> <td style="width: 10%;">Area = -2</td> </tr> </table> <p style="font-size: small;">Students should then understand that the expansion is by looking inside the grid</p> <p style="font-size: small;">Notice that the sign is always included even if positive to emphasise the multiplication of a positive or negative number.</p>	x	2x	-1	2	Area = 4x	Area = -2	<p style="font-size: small;">Students should then be able to expand a single bracket without the need to associate the size of each segment to the value.</p> $\begin{array}{r l} x & 2x \quad -1 \\ 2 & 4x \quad -2 \\ \hline & = 4x - 2 \end{array}$	
x	2x	-1							
2	Area = 4x	Area = -2							

Expand  $2y(x + y + 1)$

	<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="width: 10%;">x</td> <td style="width: 20%;">x</td> <td style="width: 20%;">+y</td> <td style="width: 10%;">+1</td> </tr> <tr> <td style="width: 10%;">2y</td> <td style="width: 20%;">Area = 2xy</td> <td style="width: 20%;">Area = +2y<sup>2</sup></td> <td style="width: 10%;">Area = +2y</td> </tr> </table> <p style="text-align: center;">= 2xy + 2y<sup>2</sup> + 2y</p>	x	x	+y	+1	2y	Area = 2xy	Area = +2y <sup>2</sup>	Area = +2y	$\begin{array}{r l} x & x \quad +y \quad +1 \\ 2y & 2xy \quad +2y^2 \quad +2y \\ \hline & = 2xy + 2y^2 + 2y \end{array}$	
x	x	+y	+1								
2y	Area = 2xy	Area = +2y <sup>2</sup>	Area = +2y								

## FACTORISING SINGLE BRACKETS

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
Students should be able to use <b>algebra tiles</b> to represent an area model.	Students should be able to link factorising to the areas of rectangles. This should be represented in an adapted grid method that illustrates different variables as different sizes.	Students should extend their knowledge in the Concrete stage to remove the idea of lengths whilst still associating the working to area if required	Students should not need to use a grid. students at this stage should be able to determine the HCF of the terms and determine the final answer without working.
<b>Example(s)</b>	<b>Example(s) - These should be seen in books</b>		

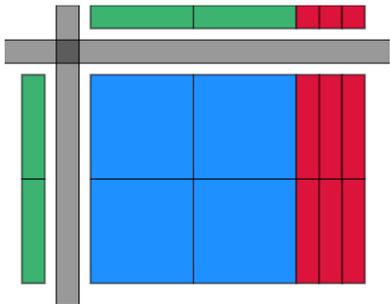
Factorise  $4x^2 - 6x$

Students should arrange Algebra tiles in such a way that they form a perfect rectangle.



Students should then understand from expanding that the divisor goes outside of the bracket (the height of the diagram) and the quotient goes inside of the bracket (the width of the diagram).

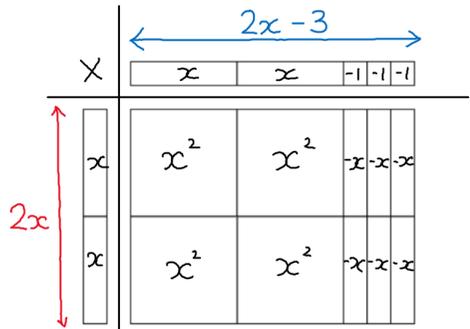
Students should use additional tiles to identify this. It may be useful for students to draw a line along the left side and top to help distinguish the tiles.



Students should then be able to state  
 $4x^2 - 6x = 2x(2x - 3)$

Students should draw the tiles in their books, using the same ideas as the Physical Step.

At this stage their sizes should be the same as the physical tiles.



$$4x^2 - 6x = 2x(2x - 3)$$

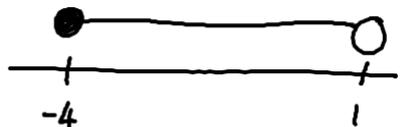
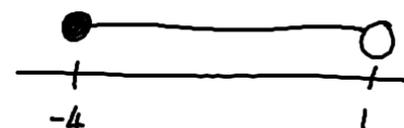
Students should represent the expression in a grid, but without the need for it to be a scaled diagram.

$$\begin{array}{r|l} x & 2x - 3 \\ \hline 2x & 4x^2 - 6x \\ \hline & = 2x(2x - 3) \end{array}$$

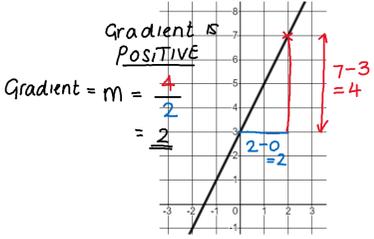
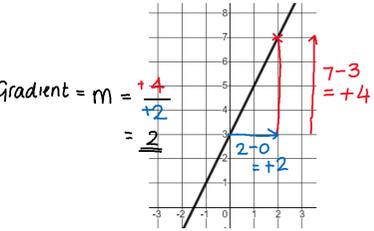
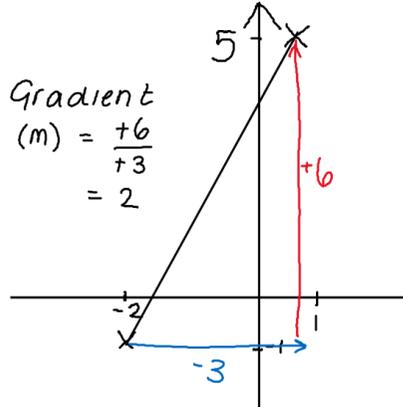
Students should be able to state their factorised answer without the need to use a diagram.

$$4x^2 - 6x = 2x(2x - 3)$$

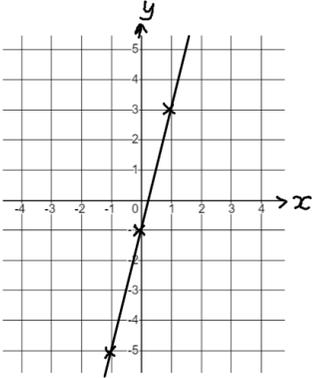
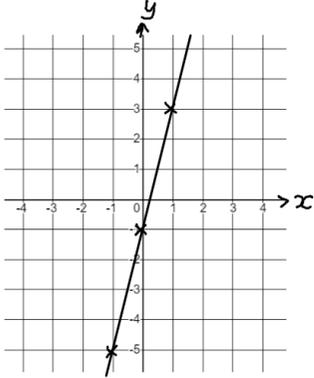
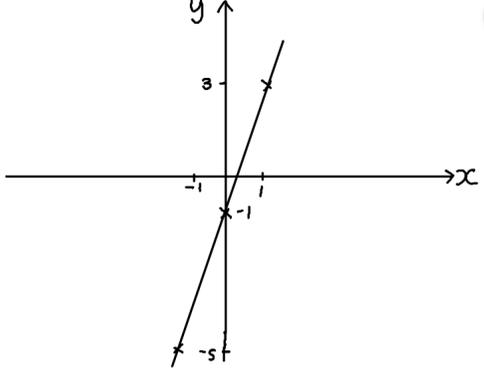
## SOLVING INEQUALITIES

GENERAL TEACHING & LEARNING POINTS	• TBA		
Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
N/A	N/A	Students should split the inequality into two parts, solving each separately.	Students should be able to apply the same operation to all three parts of the inequality, therefore removing the need to split the inequality into two parts.
Example(s)	Example(s) - These should be seen in books		
Solve the inequality $-3 \leq 2x + 5 < 7$ . Represent your solution on a number line.			
<b>NOT APPLICABLE</b>	<b>NOT APPLICABLE</b>	<p>Students should use their solving equations skills to be able to perform similar operations.</p> <p>NOTE: The inequality symbols should be left in throughout and not replaced with an = sign.</p> $  \begin{array}{l}  -3 \leq 2x + 5 < 7 \\  -5 \left( \begin{array}{l} -3 \leq 2x + 5 \\ -8 \leq 2x \end{array} \right) \begin{array}{l} -5 \\ -2 \end{array} \left  \begin{array}{l} 2x + 5 < 7 \\ 2x < 2 \end{array} \right. \begin{array}{l} -5 \\ -2 \end{array} \\  \div 2 \left( \begin{array}{l} -4 \leq x \\ -4 \leq x \end{array} \right) \begin{array}{l} \div 2 \\ \div 2 \end{array} \left  \begin{array}{l} x < 1 \\ x < 1 \end{array} \right. \begin{array}{l} \div 2 \\ \div 2 \end{array}  \end{array}  $ <p>Students should then combine their answers to form the final solution set</p> $-4 \leq x < 1$ <p>Students should then represent their answer on a number line as specified in the question.</p> 	<p>At the Abstract Stage, students should not need to separate the two parts of the inequality, instead they should apply the inverse operations to all three parts of the inequality equation.</p> $  \begin{array}{l}  -3 \leq 2x + 5 < 7 \\  -5 \left( \begin{array}{l} -3 \leq 2x + 5 \\ -8 \leq 2x \end{array} \right) \begin{array}{l} -5 \\ -5 \end{array} \left  \begin{array}{l} 2x + 5 < 7 \\ 2x < 2 \end{array} \right. \begin{array}{l} -5 \\ -5 \end{array} \\  \div 2 \left( \begin{array}{l} -4 \leq x \\ -4 \leq x \end{array} \right) \begin{array}{l} \div 2 \\ \div 2 \end{array} \left  \begin{array}{l} x < 1 \\ x < 1 \end{array} \right. \begin{array}{l} \div 2 \\ \div 2 \end{array}  \end{array}  $ <p>Students should then represent their answer on a number line as specified in the question.</p> 

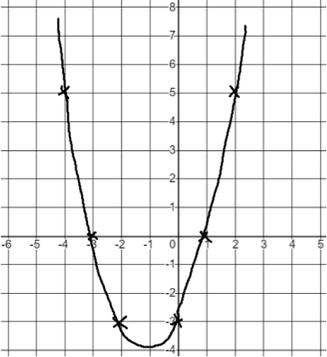
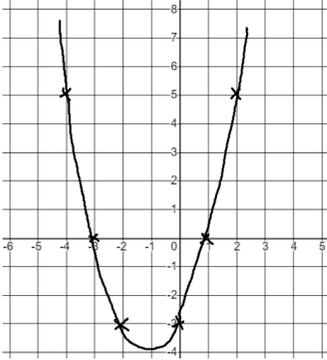
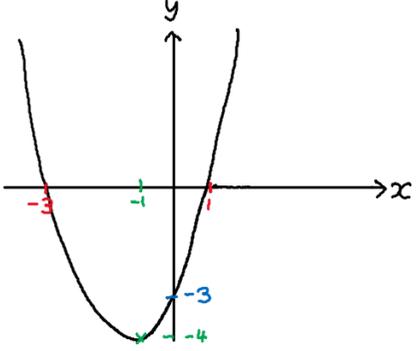
## CALCULATING THE GRADIENT OF A STRAIGHT LINE

GENERAL TEACHING & LEARNING POINTS	• TBA		
Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
N/A	Students should be able to calculate the gradient from a graph.	Students should be able to calculate the gradient from two points and drawing a sketch.	Students should be able to calculate the gradient from two points and drawing a sketch.
Example(s)	Example(s) - These should be seen in books		
	Determine the gradient of the straight line below	Calculate the gradient of the line that passes through (-2,-1) and (1,5)	
NOT APPLICABLE	<p>At this stage students work out if the gradient is positive or negative, work out the absolute value of the gradient and add in the - sign afterwards if needed.</p>  <p style="text-align: center;">Higher attaining students will be able reduce this to:</p> 	<p>At this stage the problem has been represented as a diagram, similar to that of the Pictorial Stage.</p> <p>This has been represented on an axis, however, this is not necessary. A simple sketch would suffice.</p> 	<p>At this stage students should use the <math>\frac{\text{Change in } y}{\text{Change in } x} = \frac{y_2 - y_1}{x_2 - x_1}</math> formula.</p> $(-2, -1), (1, 5)$ $x_1 \quad y_1 \quad x_2 \quad y_2$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{5 - -1}{1 - -2}$ $= \frac{6}{3}$ $= 2$

## PLOTTING LINEAR GRAPHS

GENERAL TEACHING & LEARNING POINTS	• TBA																										
Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract																								
N/A	Students should draw an accurate graph on an axis.	Students should draw an accurate graph on an axis.	At this stage students should be able to sketch the graph, indicating any intersection points with the axes.																								
<b>Example(s)</b>	<b>Example(s) - These should be seen in books</b>																										
Plot the graph of $y = 4x - 1$																											
<h1>NOT APPLICABLE</h1>	<p>Students should form a table of values.</p> <p>Students should use substitution only in order to find each co-ordinate.</p> $y = 4x - 1$ <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="border-right: 1px solid black; padding: 5px;"><math>x</math></td> <td style="padding: 5px;">-2</td> <td style="padding: 5px;">-1</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">2</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;"><math>y</math></td> <td style="padding: 5px;">-9</td> <td style="padding: 5px;">-5</td> <td style="padding: 5px;">-1</td> <td style="padding: 5px;">3</td> <td style="padding: 5px;">7</td> </tr> </table> <p style="text-align: center; margin-top: 10px;"> <span style="color: red;">(-2, -9) ↓</span>    <span style="color: red;">(0, -1) ↓</span>    <span style="color: red;">(2, 7)</span>  <span style="color: red;">(-1, -5)</span>    <span style="color: red;">(1, 3)</span> </p> 	$x$	-2	-1	0	1	2	$y$	-9	-5	-1	3	7	<p>Students should use a table of values, however, starting at the x=0 term.</p> <p>Students should then understand that the gradient is 4 and therefore for each the y value will increase by 4 for each increase of 1 in x.</p> $y = 4x - 1$ <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="border-right: 1px solid black; padding: 5px;"><math>x</math></td> <td style="padding: 5px;">-2</td> <td style="padding: 5px;">-1</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">2</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;"><math>y</math></td> <td style="padding: 5px;">-9</td> <td style="padding: 5px;">-5</td> <td style="padding: 5px;">-1</td> <td style="padding: 5px;">3</td> <td style="padding: 5px;">7</td> </tr> </table> <p style="text-align: center; margin-top: 10px;"> <span style="color: green;">+4</span>    <span style="color: green;">+4</span>    <span style="color: green;">+4</span>    <span style="color: green;">+4</span> </p> <p>At this stage students should not need to explicitly write each co-ordinate to be plotted.</p> 	$x$	-2	-1	0	1	2	$y$	-9	-5	-1	3	7	<p>Students should first plot the y-intercept, followed by increasing/decreasing by the gradient for every increase in one unit of x.</p> 
$x$	-2	-1	0	1	2																						
$y$	-9	-5	-1	3	7																						
$x$	-2	-1	0	1	2																						
$y$	-9	-5	-1	3	7																						

## PLOTTING QUADRATIC GRAPHS

GENERAL TEACHING & LEARNING POINTS	• TBA																																																										
Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract																																																								
N/A	Students should be able to plot a quadratic curve using a detailed table of values.	Students should be able to plot a quadratic curve using a two-row table of values.	Students should be able to plot a quadratic using full algebraic methods, including completing the square and factorising and solving.																																																								
Example(s)	Example(s) - These should be seen in books																																																										
	Plot the graph of $y = x^2 + 2x - 3$ . State the co-ordinates of any points of intersection with the axes.																																																										
NOT APPLICABLE	<p>Students should form a table of values where each term in the equation is separate</p> <p>Students should then add each part to form a co-ordinate.</p> $y = x^2 + 2x - 3$ <table style="margin-left: auto; margin-right: auto;"> <tr><td style="border-right: 1px solid black; padding: 2px 5px;"><math>x</math></td><td style="padding: 2px 5px;">-4</td><td style="padding: 2px 5px;">-3</td><td style="padding: 2px 5px;">-2</td><td style="padding: 2px 5px;">-1</td><td style="padding: 2px 5px;">0</td><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">2</td></tr> <tr><td style="border-right: 1px solid black; padding: 2px 5px;"><math>x^2</math></td><td style="padding: 2px 5px;">16</td><td style="padding: 2px 5px;">9</td><td style="padding: 2px 5px;">4</td><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">0</td><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">4</td></tr> <tr><td style="border-right: 1px solid black; padding: 2px 5px;">+2x</td><td style="padding: 2px 5px;">-8</td><td style="padding: 2px 5px;">-6</td><td style="padding: 2px 5px;">-4</td><td style="padding: 2px 5px;">-2</td><td style="padding: 2px 5px;">0</td><td style="padding: 2px 5px;">+2</td><td style="padding: 2px 5px;">+4</td></tr> <tr><td style="border-right: 1px solid black; padding: 2px 5px;">-3</td><td style="padding: 2px 5px;">-3</td></tr> <tr><td style="border-right: 1px solid black; padding: 2px 5px;">y</td><td style="padding: 2px 5px;">5</td><td style="padding: 2px 5px;">0</td><td style="padding: 2px 5px;">-3</td><td style="padding: 2px 5px;">-2</td><td style="padding: 2px 5px;">-3</td><td style="padding: 2px 5px;">0</td><td style="padding: 2px 5px;">5</td></tr> </table> 	$x$	-4	-3	-2	-1	0	1	2	$x^2$	16	9	4	1	0	1	4	+2x	-8	-6	-4	-2	0	+2	+4	-3	-3	-3	-3	-3	-3	-3	-3	y	5	0	-3	-2	-3	0	5	<p>Students will be able to form a table of values with just x and y rows.</p> <table style="margin-left: auto; margin-right: auto;"> <tr><td style="border-right: 1px solid black; padding: 2px 5px;"><math>x</math></td><td style="padding: 2px 5px;">-4</td><td style="padding: 2px 5px;">-3</td><td style="padding: 2px 5px;">-2</td><td style="padding: 2px 5px;">-1</td><td style="padding: 2px 5px;">0</td><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">2</td></tr> <tr><td style="border-right: 1px solid black; padding: 2px 5px;">y</td><td style="padding: 2px 5px;">5</td><td style="padding: 2px 5px;">0</td><td style="padding: 2px 5px;">-3</td><td style="padding: 2px 5px;">-2</td><td style="padding: 2px 5px;">-3</td><td style="padding: 2px 5px;">0</td><td style="padding: 2px 5px;">5</td></tr> </table> <p>Students will then be able to plot the graph from this.</p> 	$x$	-4	-3	-2	-1	0	1	2	y	5	0	-3	-2	-3	0	5	<p>Students should be able to sketch the graph by using a combination of factorising to determine the roots, completing the square and intercepts.</p> $y = x^2 + 2x - 3$ <p style="text-align: right;"><i>y-intercept</i></p> <p style="color: red;">Finding Roots:</p> $x^2 + 2x - 3 = 0$ $(x - 1)(x + 3) = 0$ $x = 1 \quad x = -3$ <p style="color: green;">Finding Turning Point</p> $(x + 1)^2 - 1 - 3$ $= (x + 1)^2 - 4$ $TP = (-1, -4)$ 
$x$	-4	-3	-2	-1	0	1	2																																																				
$x^2$	16	9	4	1	0	1	4																																																				
+2x	-8	-6	-4	-2	0	+2	+4																																																				
-3	-3	-3	-3	-3	-3	-3	-3																																																				
y	5	0	-3	-2	-3	0	5																																																				
$x$	-4	-3	-2	-1	0	1	2																																																				
y	5	0	-3	-2	-3	0	5																																																				

**Curriculum Area:**

# **RATIO AND PROPORTION**

**Converting Units**  
**Writing Ratios as Fractions**  
**Sharing in a Ratio**  
**Compound Measures**  
**Direct Proportion**  
**Best Buy Problems**  
**Exchange Rates**

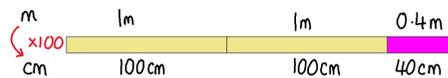
## CONVERTING UNITS

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
Students could use Cuisenaire Rods (base 10) to represent the original length.	Students should use a bar model to represent a similar idea to that of the Physical Stage.	Students should use a proportion table to scale up or down each quantity.	Students should be able to state the final conversion without any working.
<b>Example(s)</b>	<b>Example(s) - These should be seen in books</b>		

Convert 2.4m into cm.

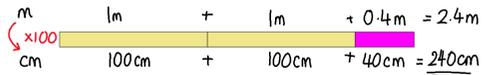
It will be beneficial if students use the manipulatives on top of a whiteboard for this stage.

Students should represent the 2.4m using two '10' rods and one '4' rod.



Notice the scale factor has been indicated on the left hand side.

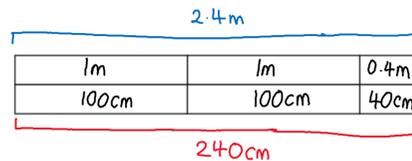
Students should then sum above and below.



Students should then state their final answer.

$$2.4\text{ m} = 240\text{ cm}$$

Students should represent the problem as a bar model clearly equating each part of the bar model in both units, as shown below.



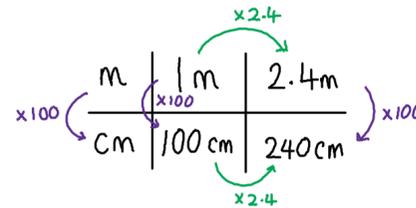
Notice, the diagram is roughly to scale (0.4m is smaller than the 1m), this emphasises that, when converted, the 0.4m cannot be more than 100cm.

Students should then state their final answer.

$$2.4\text{ m} = 240\text{ cm}$$

Students should set the work out in a proportion table.

**ALL students** should be able to perform the conversion in two different ways, shown in green and purple below.



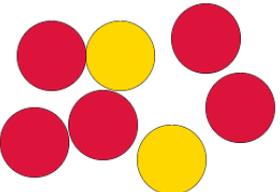
Students should then state their final answer.

$$2.4\text{ m} = 240\text{ cm}$$

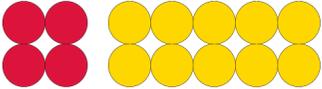
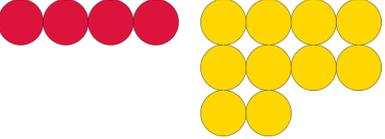
Students will be able to state the conversion immediately.

$$2.4\text{ m} = 240\text{ cm}$$

## WRITING RATIOS AS A FRACTIONS

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract																																																			
Students should be able to represent a problem using counters.	Students should be able to represent the information using a bar model.	Students should be able to use a single bar model to aid their understanding	Students should be able to write the fraction without the need for a diagram.																																																			
<b>Example(s)</b>	<b>Example(s) - These should be seen in books</b>																																																					
The ratio of yellow to red counters is 2:5. What fraction of the counters is yellow?																																																						
<p>Students should be able to represent the problem as below.</p>  <p>Notice that this clearly shows the different counters separately.</p> <p>Students should then physically move the counters together, this is important so that students understand that in total there are 7 parts.</p>  <p>Students should then apply their understanding of what a fraction is to identify that 5 out of the 7 counters are red. Students should clearly write this.</p> <p style="font-size: 1.2em;">2 out of 7 are yellow</p> <p style="font-size: 1.5em;"><math>\frac{2}{7}</math> are yellow</p>	<p>Students should represent this as a bar model, with colour side-by-side.</p> <table border="1" style="margin: 0 auto; text-align: center;"> <tr> <td>Y</td><td>Y</td><td>:</td><td>R</td><td>R</td><td>R</td><td>R</td><td>R</td> </tr> </table> <p>Students should then combine each side into a single bar to represent the total number of parts.</p> <table border="1" style="margin: 0 auto; text-align: center;"> <tr> <td>Y</td><td>Y</td><td>R</td><td>R</td><td>R</td><td>R</td><td>R</td> </tr> </table> <p>Students should then add the fraction of each colour underneath their representation.</p> <table border="1" style="margin: 0 auto; text-align: center;"> <tr> <td>Y</td><td>Y</td><td>R</td><td>R</td><td>R</td><td>R</td><td>R</td> </tr> <tr> <td colspan="2" style="border: none;">┌──────────┐</td> <td colspan="5" style="border: none;">└──────────┘</td> </tr> <tr> <td colspan="2" style="border: none;"><math>\frac{2}{7}</math></td> <td colspan="5" style="border: none;"><math>\frac{5}{7}</math></td> </tr> </table> <p style="text-align: center; font-size: 1.5em;"><math>\frac{2}{7}</math> are yellow</p>	Y	Y	:	R	R	R	R	R	Y	Y	R	R	R	R	R	Y	Y	R	R	R	R	R	┌──────────┐		└──────────┘					$\frac{2}{7}$		$\frac{5}{7}$					<p>Students should understand the idea of a ratio by this stage.</p> <p>Students should therefore be able to draw the diagram below:</p> <table border="1" style="margin: 0 auto; text-align: center;"> <tr> <td>Y</td><td>Y</td><td>R</td><td>R</td><td>R</td><td>R</td><td>R</td> </tr> </table> <p>Students should then be able to state the appropriate fraction from this diagram alone.</p> <p style="text-align: center; font-size: 1.5em;"><math>\frac{2}{7}</math> are yellow</p>	Y	Y	R	R	R	R	R	<p>Students should be able to represent the problem using a table. Consistency is important here as this layout will be used regularly during sharing in a ratio.</p> <table style="margin: 0 auto; text-align: center;"> <tr> <td>Yellow</td><td>:</td><td>Red</td><td style="border-left: 1px solid black; padding-left: 10px;">Total Parts</td> </tr> <tr> <td>2</td><td>:</td><td>5</td><td style="border-left: 1px solid black; padding-left: 10px;"><math>2 + 5 = 7</math></td> </tr> </table> <p style="text-align: center; font-size: 1.5em;"><math>\frac{2}{7}</math> are yellow</p>	Yellow	:	Red	Total Parts	2	:	5	$2 + 5 = 7$
Y	Y	:	R	R	R	R	R																																															
Y	Y	R	R	R	R	R																																																
Y	Y	R	R	R	R	R																																																
┌──────────┐		└──────────┘																																																				
$\frac{2}{7}$		$\frac{5}{7}$																																																				
Y	Y	R	R	R	R	R																																																
Yellow	:	Red	Total Parts																																																			
2	:	5	$2 + 5 = 7$																																																			

# SIMPLIFYING RATIO

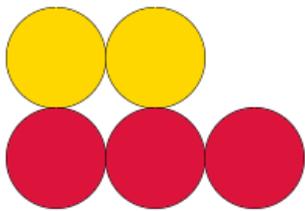
Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract																																																												
Students should be able to represent a problem using manipulatives (counters or multi-link cubes).	Students should be able to represent the information using a 'comparative' bar model and a 'cumulative' bar model.	Students should use a 'comparative' bar model and use abstract knowledge that each part is of equal value, thus using division.	Students should be able to share the ratio without needing a diagram. Students can then use equality of ratios and proportions.																																																												
<b>Example(s)</b>	<b>Example(s) - These should be seen in books</b>																																																														
Simplify the ratio 4:10.																																																															
<p>Students should be able to represent the ratio using counters.</p>  <p>Students should then attempt to re-arrange the counters into the same number of rows, two in this case.</p>  <p>Students should apply their understanding of what a ratio is to remove all rows of counters except the first.</p>  <p>Students should then be able to state that 4:10 can be simplified to 2:5.</p>	<p>Students should represent this as a 'comparative' bar model, with colour on top of each other</p> <table border="1" style="margin: 5px auto; border-collapse: collapse;"> <tr><td>Y</td><td>Y</td><td>Y</td><td>Y</td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td>R</td><td>R</td><td>R</td><td>R</td><td>R</td><td>R</td><td>R</td><td>R</td><td>R</td><td>R</td></tr> </table> <p>Students should then recognise that the diagram would be equal if two yellow parts were moved as shown:</p> <table border="1" style="margin: 5px auto; border-collapse: collapse;"> <tr><td>Y</td><td>Y</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td>R</td><td>R</td><td>R</td><td>R</td><td>R</td><td></td><td></td><td></td><td></td><td></td></tr> </table> <p>Students should apply their understanding of what a ratio is to remove all but one group and the ratio remains equal.</p> <table border="1" style="margin: 5px auto; border-collapse: collapse;"> <tr><td>Y</td><td>Y</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td>R</td><td>R</td><td>R</td><td>R</td><td>R</td><td></td><td></td><td></td><td></td><td></td></tr> </table> <p style="text-align: center;"><math>R : Y</math> <math>= 2 : 5</math></p>	Y	Y	Y	Y							R	R	R	R	R	R	R	R	R	R	Y	Y									R	R	R	R	R						Y	Y									R	R	R	R	R						<p>Students should represent the ratio in a single line.</p> <p style="text-align: center; font-size: 1.2em;">Y Y Y Y : R R R R R R R R R R</p> <p>Students should then be able to identify that each side of the ratio could be grouped into 2s. Higher attaining students could identify that this is the highest common factor of the two parts.</p> <p style="text-align: center;">Students should circle these groups.</p> <p style="text-align: center;"> <span style="border: 1px solid black; padding: 2px;">Y</span> :              <span style="border: 1px solid black; padding: 2px;">R</span> </p> <p>Students should then understand the ratio of the original number of each colour is the same ratio as the number of groups.</p> <p style="text-align: center; font-size: 1.5em;"> <math>R : Y</math>  <math>= 2 : 5</math> </p>	<p>Students should represent the ratio in a table.</p> <p>Students should clearly show that the division operations need to be applied to both sides of the ratio to keep them equivalent.</p> <p style="text-align: center; font-size: 1.5em;"> <math>R : Y</math>  <math>4 : 10</math>  <math>\div 2 \quad 2 : 5 \quad \div 2</math> </p>
Y	Y	Y	Y																																																												
R	R	R	R	R	R	R	R	R	R																																																						
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Y	Y																																																														
R	R	R	R	R																																																											
<p style="text-align: center;"><b>Discussion Point</b></p> <p>Students should be able to confidently explain why the counters cannot be arranged as below</p> 																																																															

## SHARING IN A RATIO – SHARING TOTAL

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
Students should be able to represent a problem using manipulatives (counters or multi-link cubes).	Students should be able to represent the information using a 'comparative' bar model and a 'cumulative' bar model.	Students should use a 'comparative' bar model and use abstract knowledge that each part is of equal value, thus using division.	Students should be able to share the ratio without needing a diagram. Students can then use equality of ratios and proportions.
<b>Example(s)</b>	<b>Example(s) - These should be seen in books</b>		

There are 60 counters in a box. They are either yellow or red. The ratio of yellow counters to red counters is 2:3. How many of each colour are in the box?

Students should be able to represent the problem as below.



Students should understand that the total value of all five parts is 60 and all parts represent the same amount, i.e. the manipulative below represents 60.

Students should then identify the value of one cube:



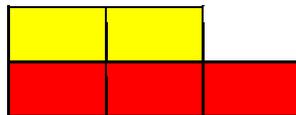
Students should then understand since there are 2 yellow and 3 red blocks this represents:

Yellow:  $12 + 12 = 24$   
 Red:  $12 + 12 + 12 = 36$

[Notice that this is represented as repeated addition and not multiplication]

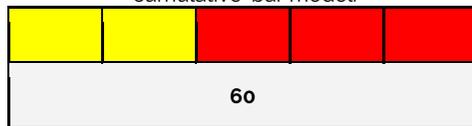
So there are 24 yellow and 36 red.

Students should be able to represent the information in the question pictorially as a 'comparative' bar model as below:



[Notice this is represented as yellow above red, this makes it easier to compare the quantities and also for more difficult problems - see later]

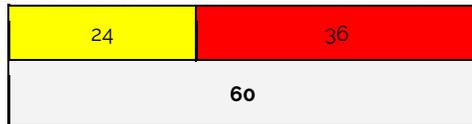
Students should understand that the total value of all five parts is 60 and represent this as a 'cumulative' bar model.



Students should then be able to identify the value of each part.

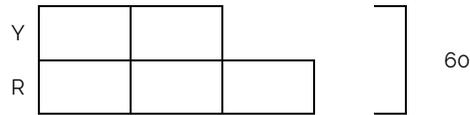


Students will then see the number of each colour.



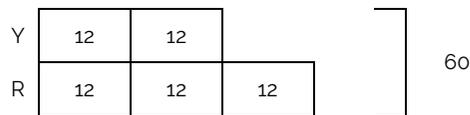
So there are 24 yellow and 36 red.

Students should be able to represent the information in the question pictorially, including the total number, as below:

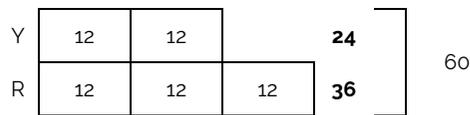


Students should then know to find the value of each part is the total divided by the number of parts.

Students should then annotate the value of each part on their diagram:



Totalling each colour



So there are 24 yellow and 36 red.

Students should be able to identify the total number of parts, then identify the scale factor to multiply to make the total parts to the total number of counters in this case.

**Note: Parts in black are step one, parts in blue are step two.**

Yellow	:	Red	Total Parts
2	:	3	5
<i>X12</i>		<i>X12</i>	<i>X12</i>
24	:	36	60

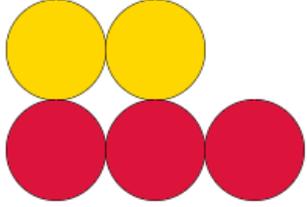
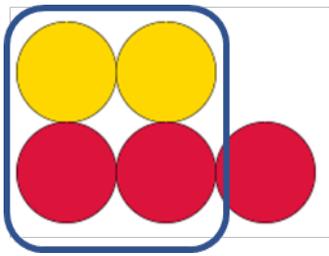
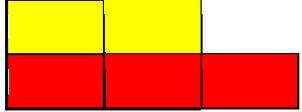
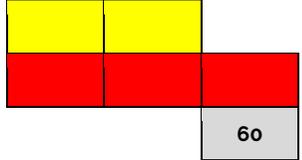
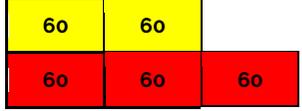
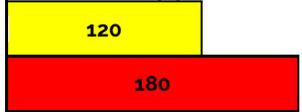
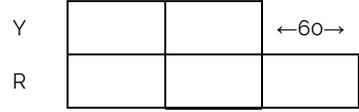
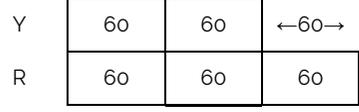
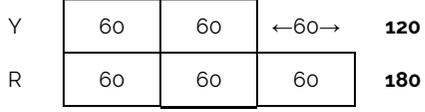
Students should then clearly state the answer either as a ratio or writing the number of each colour in this case. Which of these will depend on the exact wording of each question.

Y:R = **24:60**

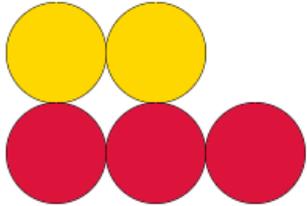
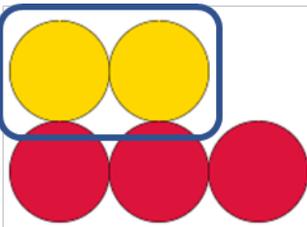
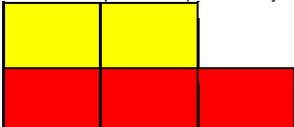
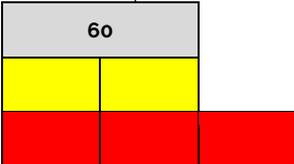
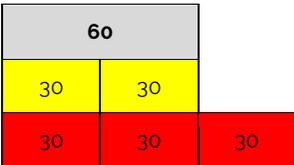
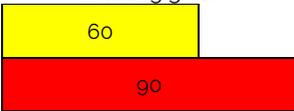
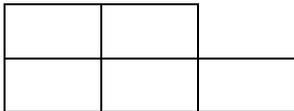
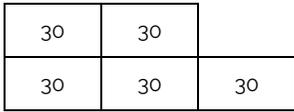
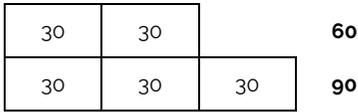
OR

Yellow = **24 counters**  
 Red = **36 counters**

## SHARING IN A RATIO - DIFFERENCES

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract												
Students should be able to represent a problem using manipulatives (counters or multi-link cubes).	Students should be able to represent the information using a 'comparative' bar model and a 'cumulative' bar model.	Students should use a 'comparative' bar model and use abstract knowledge that each part is of equal value, thus using division.	Students should be able to share the ratio without needing a diagram. Students can then use equality of ratios and proportions.												
<b>Example(s)</b>	<b>Example(s) - These should be seen in books</b>														
They are either yellow or red counters in a box. The ratio of yellow counters to red counters is 2:3. There are 60 <b>more</b> red counters than yellow. How many yellow counters and red counters were in the box?															
<p>Students should be able to represent the problem as below.</p>  <p>Students should understand that the four blocks circled have equal parts and the extra red block represents the 60 extra red counters.</p>  <p style="text-align: center;">So each block represents 60</p> <p>Students should then understand since there are 2 yellow and 3 red blocks this represents:            Yellow: <math>60 + 60 = 120</math>            Red: <math>60 + 60 + 60 = 180</math></p> <p style="text-align: center;">So there are 120 yellow and 180 red.</p>	<p>Students should be able to represent the information in the question pictorially as below:</p>  <p>[Notice this is represented as yellow above red, this makes it easier to compare the quantities and also for more difficult problems - see later]</p> <p>Students should understand that the total value of all five parts is 60 and represent this as a bar model.</p>  <p>Students should then be able to identify the value of each part.</p>  <p style="text-align: center;">Totalling gives</p>  <p style="text-align: center;">So there are 120 yellow and 180 red.</p>	<p>Students should be able to represent the information in the question pictorially, including the difference, as below:</p>  <p>Students should then understand each part must represent the same value. Students should annotate their diagram.</p>  <p>Students should then be able to total each part.</p>  <p style="text-align: center;">So there are 120 yellow and 180 red.</p>	<p>Students should be able to identify the <u>difference</u> in the number of parts, then divide to determine the amount represented by 1 part.</p> <p style="text-align: center;"><i>Note: Parts in <b>black</b> are step one, parts in <b>blue</b> are step two.</i></p> <table style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <tr> <td style="padding: 5px;">Yellow</td> <td style="padding: 5px;">:</td> <td style="padding: 5px;">Red</td> <td style="border-left: 1px solid black; padding: 5px;">Parts Difference</td> </tr> <tr> <td style="padding: 5px; text-align: center;">2</td> <td style="padding: 5px;"></td> <td style="padding: 5px; text-align: center;">3</td> <td style="border-left: 1px solid black; padding: 5px; text-align: center;">1</td> </tr> <tr> <td style="padding: 5px; text-align: center;"><i>X60</i> 120</td> <td style="padding: 5px;"></td> <td style="padding: 5px; text-align: center;"><i>X60</i> 180</td> <td style="border-left: 1px solid black; padding: 5px; text-align: center;">60 <i>X60</i></td> </tr> </table> <p>Students should then clearly state the answer either as a ratio or writing the number of each colour in this case. Which of these will depend on the exact wording of each question.</p> <p style="text-align: center;">Y:R = <b>120:180</b></p> <p style="text-align: center;">OR</p> <p style="text-align: center;">Yellow = <b>120 counters</b>            Red = <b>180 counters</b></p>	Yellow	:	Red	Parts Difference	2		3	1	<i>X60</i> 120		<i>X60</i> 180	60 <i>X60</i>
Yellow	:	Red	Parts Difference												
2		3	1												
<i>X60</i> 120		<i>X60</i> 180	60 <i>X60</i>												

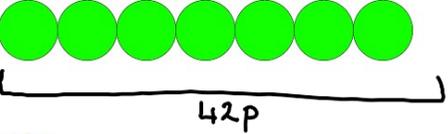
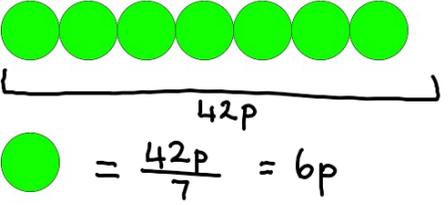
## SHARING IN A RATIO – GIVEN ONE AMOUNT

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract												
Students should be able to represent a problem using manipulatives (counters or multi-link cubes).	Students should be able to represent the information using a 'comparative' bar model and a 'cumulative' bar model.	Students should use a 'comparative' bar model and use abstract knowledge that each part is of equal value, thus using division.	Students should be able to share the ratio without needing a diagram. Students can then use equality of ratios and proportions.												
<b>Example(s)</b>	<b>Example(s) - These should be seen in books</b>														
<p>They are either yellow or red counters in a box. The ratio of yellow counters to red counters is 2:3. There are 60 yellow counters. How many counters are in the box in <b>total</b>?</p>															
<p>Students should be able to represent the problem as below.</p>  <p>Students should understand that the yellow blocks represent 60.</p>  <p>So each block represents 30.</p> <p>Students should then understand that the 3 red blocks this represents:</p> <p style="text-align: center;">Red: <math>30 + 30 + 30 = 90</math></p> <p style="text-align: center;">So there are <math>60 + 90 = 150</math> in total.</p>	<p>Students should be able to represent the information in the question pictorially as below:</p>  <p>[Notice this is represented as yellow above red, this makes it easier to compare the quantities and also for more difficult problems - see later!]</p> <p>Students should understand that the total value of all five parts is 60 and represent this as a bar model.</p>  <p>Students should then be able to identify the value of each part.</p>  <p style="text-align: center;">Totalling gives</p>  <p style="text-align: center;"><b><math>60+90 = 150</math> counters in total</b></p>	<p>Students should be able to represent the information in the question pictorially, including the total yellow counters., as below:</p>  <p>Students should then understand each part must represent the same value. Students should annotate their diagram.</p>  <p>Students should then be able to total each part and the overall total.</p>  <p style="text-align: center;"><b><math>60+90 = 150</math> counters in total</b></p>	<p>Students should be able to identify the number of parts represented by the given quantity, then divide to determine the amount represented by 1 part.</p> <p style="text-align: center;"><i>Note: Parts in <b>black</b> are step one, parts in <b>blue</b> are step two.</i></p> <table style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <tr> <td style="padding: 0 10px;">Yellow</td> <td style="padding: 0 10px;">:</td> <td style="padding: 0 10px;">Red</td> <td style="border-left: 1px solid black; padding: 0 10px;">Total Parts</td> </tr> <tr> <td style="text-align: center; color: black;">2</td> <td style="text-align: center;">:</td> <td style="text-align: center; color: black;">3</td> <td style="border-left: 1px solid black; text-align: center; color: black;">5</td> </tr> <tr> <td style="text-align: center; color: blue;">60</td> <td style="text-align: center;">:</td> <td style="text-align: center; color: blue;">90</td> <td style="border-left: 1px solid black; text-align: center; color: blue;">150</td> </tr> </table> <p style="text-align: center;">Students should then clearly state the answer either as a ratio or writing the number of each colour in this case. Which of these will depend on the exact wording of each question.</p> <p style="text-align: center;">Y:R = 60:90</p> <p style="text-align: center;">Total = <math>60 + 90 = 150</math></p>	Yellow	:	Red	Total Parts	2	:	3	5	60	:	90	150
Yellow	:	Red	Total Parts												
2	:	3	5												
60	:	90	150												

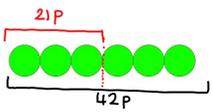
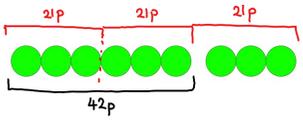
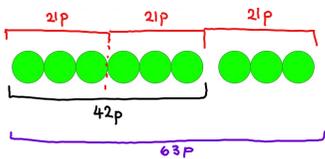
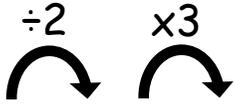
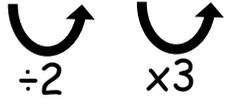
## COMPOUND MEASURES

GENERAL TEACHING & LEARNING POINTS	<ul style="list-style-type: none"> <li>Students should have a secure understanding of units and what these units mean. E.g. mph is "how many miles something travels per hour" or Pressure "How many Newtons per every 1 cm<sup>2</sup>"</li> <li>Students should apply their understanding of proportion when working with Compound Measures</li> <li>Try to avoid using formula triangles with Compound Measures as these do not promote teaching for understanding.</li> </ul>																						
Stage 1 <b>Physical</b>	Stage 2 <b>Pictorial</b>	Stage 3 <b>Semi-Abstract</b>	Stage 4 <b>Abstract</b>																				
N/A	Students should use a bar model to represent the problem.	Students should use a proportion table to scale up or down each quantity.	Students should confidently state and use the Compound Measures formulae without the need for an alternative representation.																				
<b>Example(s)</b>	<b>Example(s) - These should be seen in books</b>																						
A car travels at a speed of 45 mph for 20 minutes. How far does it travel in this time?																							
NOT APPLICABLE	<p>Students should represent this as a bar model.</p> <p>Notice that the distance travelled in 1 hour is represented using equal length bars.</p> <table border="1" style="margin: 5px auto; border-collapse: collapse;"> <tr><td style="text-align: center;">45 miles</td></tr> <tr><td style="text-align: center;">1 hour</td></tr> </table> <p>Students should then understand that 20 minutes is one third of an hour and therefore this needs to be split into three equal parts.</p> <p>Students should then use proportion to calculate the distance travelled in this time.</p> <table border="1" style="margin: 5px auto; border-collapse: collapse;"> <tr><td style="text-align: center;">15 miles</td><td style="text-align: center;">15 miles</td><td style="text-align: center;">15 miles</td></tr> <tr><td colspan="3" style="text-align: center;">45 miles</td></tr> <tr><td colspan="3" style="text-align: center;">1 hour</td></tr> <tr><td style="text-align: center;">20 mins</td><td style="text-align: center;">20 mins</td><td style="text-align: center;">20 mins</td></tr> </table> <p>Students should then be able to identify that in 20 minutes the car travelled 15 miles.</p>	45 miles	1 hour	15 miles	15 miles	15 miles	45 miles			1 hour			20 mins	20 mins	20 mins	<p>At this stage students should use a proportion table to determine the unknown quantity.</p> <div style="text-align: center; margin: 10px 0;"> <math>\div 3</math>   </div> <table border="1" style="margin: 5px auto; border-collapse: collapse;"> <tr><td style="text-align: center;">Distance</td><td style="text-align: center;">45 miles</td><td style="text-align: center;">15 miles</td></tr> <tr><td style="text-align: center;">Time</td><td style="text-align: center;">1 hour</td><td style="text-align: center;"><math>\frac{1}{3}</math> hour</td></tr> </table> <div style="text-align: center; margin: 10px 0;"> <math>\div 3</math>   </div>	Distance	45 miles	15 miles	Time	1 hour	$\frac{1}{3}$ hour	<p>Students should use and substitute into the formula.</p> <p>Avoid using the formula triangles, however, this should be the only stage at which then could be used if students are unable to understand the basic concepts after significant time has been spent.</p> <p>Students should clearly state the formula they are using (in its original form) followed by any re-arrangement as a separate stage of working.</p> <p>Students should state the value of quantities in the question, with particular emphasis on units.</p> <p style="text-align: center;">S = 45 mph    t = <math>\frac{1}{3}</math> hour</p> <p style="text-align: center;">Speed = <math>\frac{\text{Distance}}{\text{Time}}</math></p> <p style="text-align: center;">Distance = Speed x Time</p> <p style="text-align: center;">Distance = <math>45 \times \frac{1}{3} = \mathbf{15 \text{ miles}}</math></p>
45 miles																							
1 hour																							
15 miles	15 miles	15 miles																					
45 miles																							
1 hour																							
20 mins	20 mins	20 mins																					
Distance	45 miles	15 miles																					
Time	1 hour	$\frac{1}{3}$ hour																					

# DIRECT PROPORTION

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract																																																																						
Students should be able to represent this problem using counters or multi-link cubes.	Students should be able to represent the information using a bar model in their books.	Students should use a direct proportion table to be able to represent and solve the problem	Students should be able to use direct proportion using the same format as simplifying a ratio.																																																																						
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7 pens cost 42p. What is the cost of 1 pen?																																																																									
<p>Students should represent the problem by using 7 counters or cubes to represent the 7 pens.</p>  <p>Students should then be able to work out the value of 1 pen.</p> <p>Higher attaining students will be able to spot that, for other problems, that it isn't always necessary to find the value of 1 part first. E.g. 8 pens cost 80p work out the cost of 2 pens. Students will be able to divide by 4 in one step rather than dividing by 8 then multiplying by 2.</p>  <p>Students should then clearly answer the question.</p> <p style="text-align: center; font-size: 1.2em;">1 pen = 6p</p>	<p>Students should draw the diagram below in their books.</p> <table border="1" style="margin: 0 auto; border-collapse: collapse; text-align: center;"> <tr> <td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td> </tr> <tr> <td colspan="8">42p</td> </tr> </table> <p>Students should then clearly show that they have divided the 42p into 7 parts by adding this to their diagram and showing the division calculation.</p> <table border="1" style="margin: 0 auto; border-collapse: collapse; text-align: center;"> <tr> <td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td> </tr> <tr> <td colspan="8">42p</td> </tr> <tr> <td>6p</td><td>6p</td><td>6p</td><td>6p</td><td>6p</td><td>6p</td><td>6p</td><td>6p</td> </tr> </table> <p style="text-align: center;"><math>\frac{42p}{7} = 6p</math></p> <p>Students could then circle the part of their diagram that gives the value of 1 pen.</p> <table border="1" style="margin: 0 auto; border-collapse: collapse; text-align: center;"> <tr> <td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td> </tr> <tr> <td colspan="8">42p</td> </tr> <tr> <td>6p</td><td>6p</td><td>6p</td><td>6p</td><td>6p</td><td>6p</td><td>6p</td><td>6p</td> </tr> </table> <p style="text-align: center;"><math>\frac{42p}{7} = 6p</math></p> <p>Students should then clearly answer the question.</p> <p style="text-align: center; font-size: 1.2em;">1 pen = 6p</p>	1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	42p								1 Pen	42p								6p	1 Pen	42p								6p	<p>At this stage students should use a proportion table to determine the unknown quantity.</p> <div style="text-align: center; margin-bottom: 10px;"> <math>\div 7</math>   </div> <table border="1" style="margin: 0 auto; border-collapse: collapse; text-align: center;"> <tr> <td>Pens</td><td>7 pens</td><td>1 pen</td> </tr> <tr> <td>Cost</td><td>42p</td><td>6p</td> </tr> </table> <div style="text-align: center; margin-top: 10px;">   <math>\div 7</math> </div> <p>Students should then clearly answer the question.</p> <p style="text-align: center; font-size: 1.5em;">1 pen = 6p</p>	Pens	7 pens	1 pen	Cost	42p	6p	<p>Students should represent the problem in a table.</p> <p>Students should clearly show that the division operations need to be applied to both sides to represent the direct proportionality.</p> <p style="text-align: center; margin: 10px 0;">Pens : Cost</p> <p style="text-align: center; margin: 10px 0;">7 pens : 42p</p> <p style="text-align: center; margin: 10px 0;"><math>\div 7</math> 1 pen : 6p <math>\div 7</math></p> <p>Students should then clearly answer the question.</p> <p style="text-align: center; font-size: 1.5em;">1 pen = 6p</p>																												
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## BEST BUY PROBLEMS

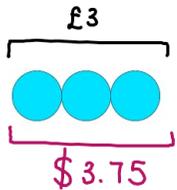
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<p>Students should represent the problem by using 7 counters or cubes to represent the 7 pens.</p> <p>Students should then be able to work out the value of 3 pens.</p>  <p>Students could use a unitary approach here, however, students should be encouraged to be efficient with their method and therefore since 9 is a multiple of 3 this would be more effective.</p> <p>Students should then add in the additional counters to their diagram in order to work out the cost of an equivalent number of pens as Shop B but from Shop A.</p>  <p>Students should then work out the cost from Shop A</p>  <p>Students should then clearly answer the question.</p> <p style="text-align: center; font-size: 1.2em;">Bella should buy from Shop A</p>	<p>Students should draw similar diagrams to the Physical Stage below in their books, this time using a bar model</p> <table border="1" style="margin-bottom: 10px; width: 100%; text-align: center;"> <tr><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td></tr> <tr><td colspan="6">42p</td></tr> </table> <table border="1" style="margin-bottom: 10px; width: 100%; text-align: center;"> <tr><td colspan="3">21p</td><td colspan="3">21p</td></tr> <tr><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td></tr> <tr><td colspan="6">42p</td></tr> </table> <table border="1" style="margin-bottom: 10px; width: 100%; text-align: center;"> <tr><td colspan="2">21p</td><td colspan="2">21p</td><td colspan="2">21p</td></tr> <tr><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td></tr> <tr><td colspan="4">42p</td><td colspan="2">21p</td></tr> </table> <table border="1" style="margin-bottom: 10px; width: 100%; text-align: center;"> <tr><td colspan="2">21p</td><td colspan="2">21p</td><td colspan="2">21p</td></tr> <tr><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td><td>1 Pen</td></tr> <tr><td colspan="4">42p</td><td colspan="2">21p</td></tr> <tr><td colspan="6">63p</td></tr> </table> <p>Students should then clearly answer the question.</p> <p style="text-align: center; font-size: 1.2em;">Bella should buy from Shop A</p>	1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	42p						21p			21p			1 Pen	42p						21p		21p		21p		1 Pen	42p				21p		21p		21p		21p		1 Pen	42p				21p		63p						<p>At this stage students should use a proportion table to determine the cost of pens in Shop A.</p> <div style="text-align: center; margin-bottom: 10px;"> <math>\div 2</math>      <math>\times 3</math>   </div> <table border="1" style="margin-bottom: 10px; width: 100%; text-align: center;"> <tr> <th style="width: 15%;">Shop A</th> <th style="width: 20%;">Pens</th> <th style="width: 20%;">6 pens</th> <th style="width: 20%;">3 pens</th> <th style="width: 25%;">9 pens</th> </tr> <tr> <th>Cost</th> <td>42p</td> <td>21p</td> <td>63p</td> <td></td> </tr> </table> <div style="text-align: center; margin-bottom: 10px;"> <math>\div 2</math>      <math>\times 3</math>   </div> <p style="text-align: center; font-size: 1.2em;">Bella should buy from Shop A</p>	Shop A	Pens	6 pens	3 pens	9 pens	Cost	42p	21p	63p		<p>Students should represent the problem in a table.</p> <p>Students should clearly show that the division operations need to be applied to both sides to represent the direct proportionality.</p> <div style="text-align: center; margin-bottom: 10px;"> <u>Shop A</u> </div> <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 5px;">Pens</td> <td style="padding: 5px;">:</td> <td style="padding: 5px;">Cost</td> <td></td> </tr> <tr> <td style="padding: 5px;">6 pens</td> <td style="padding: 5px;">:</td> <td style="padding: 5px;">42p</td> <td></td> </tr> <tr> <td style="padding: 5px;"><math>\div 2</math></td> <td></td> <td style="padding: 5px;">3 pens</td> <td style="padding: 5px;"><math>\div 2</math></td> </tr> <tr> <td></td> <td></td> <td style="padding: 5px;">:</td> <td></td> </tr> <tr> <td style="padding: 5px;"><math>\times 3</math></td> <td></td> <td style="padding: 5px;">9 pens</td> <td style="padding: 5px;"><math>\times 3</math></td> </tr> <tr> <td></td> <td></td> <td style="padding: 5px;">:</td> <td></td> </tr> <tr> <td></td> <td></td> <td style="padding: 5px;">63p</td> <td></td> </tr> </table> <p>Students should then clearly answer the question.</p> <p style="text-align: center; font-size: 1.2em;">Bella should buy from Shop A</p>	Pens	:	Cost		6 pens	:	42p		$\div 2$		3 pens	$\div 2$			:		$\times 3$		9 pens	$\times 3$			:				63p																
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# EXCHANGE RATES

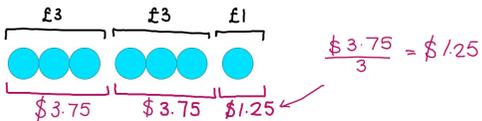
Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
Students should be able to represent this problem using counters or multi-link cubes.	Students should be able to represent the information using a bar model in their books.	Students should use a direct proportion table to be able to represent and solve the problem	Students should be able to use direct proportion using the same format as simplifying a ratio.
<b>Example(s)</b>	<b>Example(s) - These should be seen in books</b>		

The exchange rate from Pounds (£) into US Dollars (\$) is £3 = \$3.75.  
Calculate how many Dollars (\$) are equivalent to £7.

Students should be able to represent the initial problem and information with a manipulative.



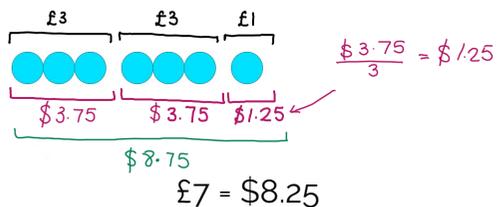
Students should then be able to scale up the given quantity, by either using a unitary method or by understanding that £7 is 2 x £3 + £1, as shown:



If a unitary method is used, students should clearly show how they obtained the value of a single unit.



Students should then be able to add up the corresponding amounts and give a final answer.



Students should draw similar diagrams to the Physical Stage below in their books, this time using a bar model.

£3
\$3.75

Students should clearly show how they are scaling up the value.

£3	£3	£1
\$3.75	\$3.75	\$1.25

$$£1 = \frac{\$3.75}{3} = \$1.25$$

Students should then show the total value and clearly state their final answer.

£3	£3	£1
\$3.75	\$3.75	\$1.25
\$8.25		

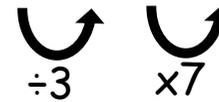
$$£7 = \$8.25$$

At this stage students should use a proportion table to determine the cost of pens in Shop A.

$\div 3$        $\times 7$

**Shop A**

£	£3	£1	£7
\$	\$3.75	\$1.25	\$8.25



Students should then clearly state their final answer.

$$£7 = \$8.25$$

Students should represent the problem in a table.

Students should clearly show that the division operations need to be applied to both sides to represent the direct proportionality.

£	:	\$
£3	:	\$3.75
$\div 3$		$\div 3$
£1	:	\$1.25
$\times 7$		$\times 7$
£7	:	\$8.25

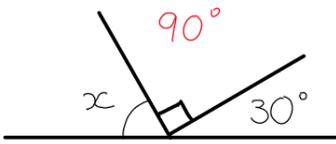
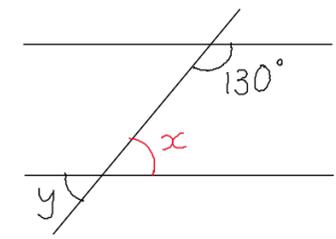
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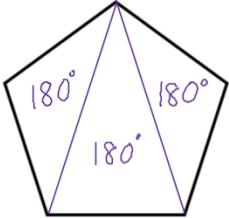
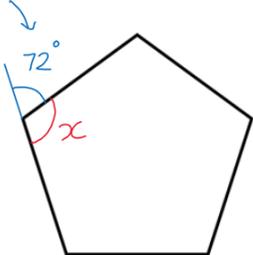
**Curriculum Area:**  
**GEOMETRY**

**Angles on Straight Lines and Points**  
**Interior and Exterior Angles**  
**Area of Polygons and Circles**  
**Surface Area**  
**Volume**  
**Transformations**

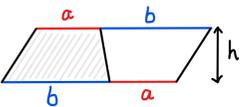
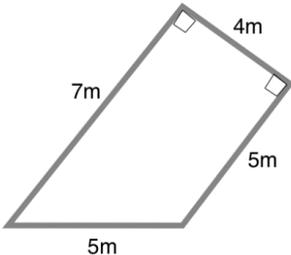
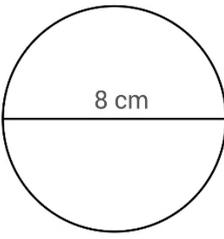
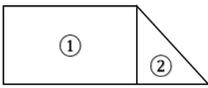
## ANGLES – STRAIGHT LINES AND POINTS

GENERAL TEACHING & LEARNING POINTS	<ul style="list-style-type: none"> <li>Students should constantly be made to justify their answers.</li> <li>Consistent terminology is essential. The terminology to be used is below.</li> </ul>											
VOCABULARY	Corresponding angles are equal.	Alternate angles are equal.	Co-interior angles add up to 180°	Vertically opposite angles are equal								
	Angles on a straight line add up to 180°		Angles around a point add up to 360°									
Stage 1 <b>Physical</b>	Stage 2 <b>Pictorial</b>	Stage 3 <b>Semi-Abstract</b>		Stage 4 <b>Abstract</b>								
Students could discover the rules for angle problems by measuring angles when diagrams are drawn to scale.	<p>Since this is a pictorial topic, these stages are combined.</p> <p>Students should form and solve an equation given angle facts.</p>											
<b>Example(s)</b>	<b>Example(s) - These should be seen in books</b>											
Students could discover the rules for angle problems by measuring angles when diagrams are drawn to scale.	<p>Students should form and solve an algebraic equation in order to determine the missing angle.</p> <p>Work out the size of angle x.</p> <div style="display: flex; align-items: center; justify-content: center;">  <div style="margin-left: 20px;"> <math display="block">x + 90^\circ + 30^\circ = 180^\circ</math> <math display="block">x + 120^\circ = 180^\circ</math> <math display="block">x = 60^\circ</math> </div> </div>											
Students could discover the rules for angle problems by measuring angles when diagrams are drawn to scale.	<p>Work out the size of angle y. Give reasons at each stage of your working.</p> <div style="display: flex; align-items: center; justify-content: center;">  <div style="margin-left: 20px;"> <table border="1" style="border-collapse: collapse;"> <thead> <tr> <th style="padding: 5px;">Working</th> <th style="padding: 5px;">Reasons</th> </tr> </thead> <tbody> <tr> <td style="padding: 5px;"><math>x + 130^\circ = 180^\circ</math></td> <td style="padding: 5px;">co-interior Angles add up to 180°</td> </tr> <tr> <td style="padding: 5px;"><math>x = 50^\circ</math></td> <td style="padding: 5px;"></td> </tr> <tr> <td style="padding: 5px;"><math>y = 50^\circ</math></td> <td style="padding: 5px;">vertically opposite angles are equal</td> </tr> </tbody> </table> </div> </div>				Working	Reasons	$x + 130^\circ = 180^\circ$	co-interior Angles add up to 180°	$x = 50^\circ$		$y = 50^\circ$	vertically opposite angles are equal
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	Students could be encouraged to split their page in half with one column as the working and the second for the reasons.											

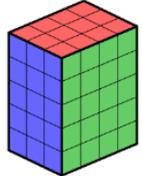
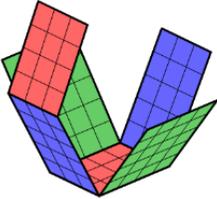
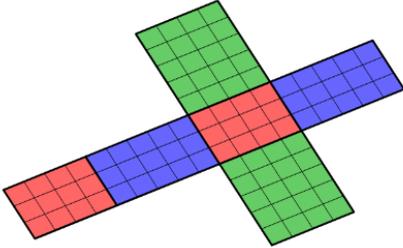
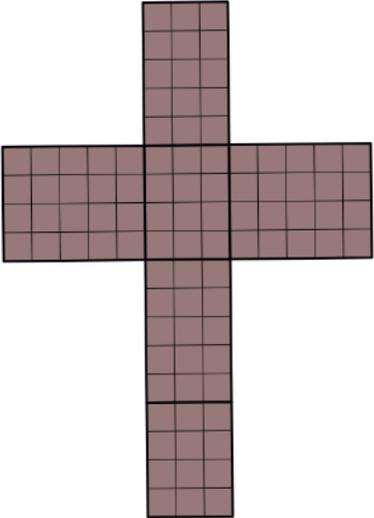
## INTERIOR AND EXTERIOR ANGLES

GENERAL TEACHING & LEARNING POINTS	<ul style="list-style-type: none"> <li>Students should not purely be taught a formula, students should understand that the volume links to the number of layers of a shape</li> <li>Students should then understand that the volume of a prism (and cylinder) is the Area of the Cross-Section x Length</li> </ul>		
Stage 1 <b>Physical</b>	Stage 2 <b>Pictorial</b>	Stage 3 <b>Semi-Abstract</b>	Stage 4 <b>Abstract</b>
Students could discover the rules for angle problems by measuring angles when diagrams are drawn to scale.	Students should determine the interior and exterior angles of a polygon by splitting the shape into triangles.	Students should understand the link between interior and exterior angles.	Students should understand and apply the formula $(n - 2) \times 180^\circ$
Example(s)	Example(s) - These should be seen in books		
Determine the sum of the interior angles of a regular pentagon.			
<p>Students could discover the rules for angle problems by measuring angles when diagrams are drawn to scale.</p>	<p>Students should pick a particular point and separate the polygon into triangles.</p>  <p style="margin-left: 40px;"> <math>\text{Pentagon} = 3 \text{ triangles}</math>  <math>= 3 \times 180^\circ</math>  <math>= 540^\circ</math> </p>	<p>Students should understand that interior and exterior angles sum to <math>180^\circ</math> and that the sum of the exterior angles of any polygon sum to <math>360^\circ</math>.</p> <p style="margin-left: 20px;"><math>\frac{360^\circ}{5} = 72^\circ</math></p>  <p style="margin-left: 40px;"> <math>x + 72^\circ = 180^\circ</math>  <math>x = 108^\circ</math> </p> <p style="margin-left: 40px;"> <math>\text{Sum of Interior Angles} = 108^\circ \times 5 = 540^\circ</math> </p>	<p style="margin-left: 20px;">Pentagon = 5 sides</p> <p style="margin-left: 20px;"> <math>\text{Sum of interior Angles} = (n - 2) \times 180^\circ</math>  <math>= (5 - 2) \times 180^\circ</math>  <math>= 3 \times 180^\circ</math>  <math>= 540^\circ</math> </p>

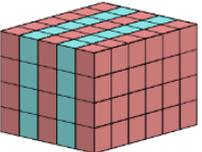
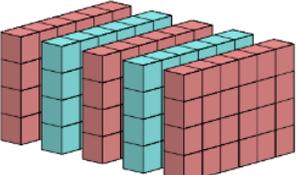
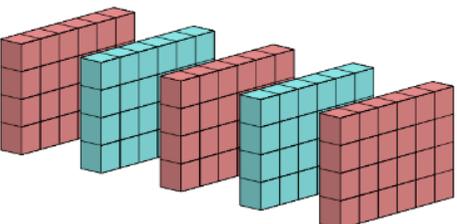
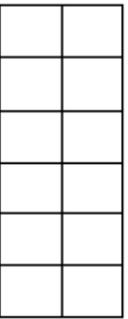
# AREA OF POLYGONS AND CIRCLES

GENERAL TEACHING & LEARNING POINTS	<ul style="list-style-type: none"> <li>Students should be taught the areas of polygons in the following order:                             <div style="display: flex; justify-content: space-around; align-items: center; margin: 5px 0;"> <span style="background-color: #FF8C00; color: white; padding: 5px 15px; border-radius: 10px;">Rectangles</span> <span style="background-color: #FF69B4; color: white; padding: 5px 15px; border-radius: 10px;">Parallelograms</span> <span style="background-color: #FF4500; color: white; padding: 5px 15px; border-radius: 10px;">Triangles</span> <span style="background-color: #FF6347; color: white; padding: 5px 15px; border-radius: 10px;">Trapeziums</span> <span style="background-color: #A9A9A9; color: white; padding: 5px 15px; border-radius: 10px;">Circles</span> </div> </li> <li>Students should NOT be merely taught a formula. Students should <b>understand</b> that each formula can be determined from the area of a <u>parallelogram or rectangle</u></li> </ul>			
Stage 1 <b>Physical</b>	Stage 2 <b>Pictorial</b>	Stage 3 <b>Semi-Abstract</b>	Stage 4 <b>Abstract</b>	
Students could use physical versions of a rectangle or parallelogram to determine the formulae for the area of each shape.	Students should be able calculate the area of a shape, including compound shapes. Students should clearly state any formula and any values given in the questions. For answers involving $\pi$ , students should always state the exact answer prior to using any rounding.			
Example(s)	Example(s) - These should be seen in books			
<p><b>Rectangle</b> <math>A = b \times h</math> OR <math>A = bh</math></p>  <hr/> <p><b>Parallelogram</b> <math>A = b \times h</math> OR <math>A = bh</math> Students should move part of the parallelogram to the opposite side to form a rectangle.</p>  <hr/> <p><b>Parallelogram</b> <math>A = \frac{b \times h}{2}</math> OR <math>A = \frac{bh}{2}</math> Students should split a parallelogram in half to form a triangle.</p>  <hr/> <p><b>Trapezium</b> <math>A = \frac{a+b}{2} \times h</math> Students should rotate two trapezia to identify the area is half a parallelogram with side length (a+b)</p> 	<p>Calculate the area of this trapezium.</p>  <hr/> <p>Calculate the area of this circle.</p> 	<p>Students should clearly state the formula to be used.  <math>A = \frac{a+b}{2} \times h</math>                      Students should state the required information from the diagram.  <math>a = 5m</math> <math>b = 7m</math> <math>h = 4m</math>                      Students should then substitute into the formula  <math>A = \frac{5+7}{2} \times 4</math>  <math>A = 6 \times 4</math>                      Students should ensure that the units are included in the final answer.  <math>A = 24m^2</math></p> <hr/> <p>Students should clearly state the formula to be used.  <math>A = \pi r^2</math>                      Students should state the required information from the diagram.  <math>d = 8cm</math> so <math>r = 4cm</math>                      Students should then substitute into the formula  <math>A = \pi \times 4^2</math>                      Students must include the answer in terms of <math>\pi</math> answer  <math>A = 16\pi</math>                      Students should ensure that the units are included in the final answer.  <math>A = 50.27cm^2</math></p>		
<p><b>When working with compound shapes, students should clearly label each part of the shape. Students should then calculate each area separately as shown above.</b></p>				

## SURFACE AREA

GENERAL TEACHING & LEARNING POINTS	<ul style="list-style-type: none"> <li>Students should be encouraged to understand what is meant by the surface area by investigating the nets of shapes.</li> <li>Students should be able to find the surface area of cubes, cuboids, prisms and cylinders.</li> </ul>																																
Stage 1 <b>Physical</b>	Stage 2 <b>Pictorial</b>	Stage 3 <b>Semi-Abstract</b>	Stage 4 <b>Abstract</b>																														
Students should use the 3D Geometric Shapes to understand the different faces.	Students should use a net to find the surface area.	Students should list each of the faces then total.	Students should identify repeated faces.																														
Example(s)	Example(s) - These should be seen in books																																
<p>A cuboid has a length of 3cm, a width of 4cm and a height of 5cm. Work out the surface area of the shape.</p>																																	
<p>Students should use the 3D Geometric Shapes to understand the different faces.</p> <p>The Maths Pad interactive tool could be used to help understanding.</p> <div style="text-align: center;">        </div> <p style="text-align: center;">Students will then be able to count the squares to find the Surface Area.</p>	<p>Students should draw a net of the cuboid using the squares in their books.</p> <div style="text-align: center;">    <p>Surface Area = 94cm<sup>2</sup></p> </div>	<p>Students should list all of the sides and calculate the areas of each.</p> <p style="text-align: center;">Students should then total these.</p> <table style="margin-left: auto; margin-right: auto;"> <tr><td>Front</td><td>5 × 4</td><td>= 20</td></tr> <tr><td>Back</td><td>5 × 4</td><td>= 20</td></tr> <tr><td>Top</td><td>4 × 3</td><td>= 12</td></tr> <tr><td>Bottom</td><td>4 × 3</td><td>= 12</td></tr> <tr><td>Left</td><td>3 × 5</td><td>= 15</td></tr> <tr><td>Right</td><td>3 × 5</td><td>= 15</td></tr> </table> <p style="text-align: center;">Surface Area = 94cm<sup>2</sup></p>	Front	5 × 4	= 20	Back	5 × 4	= 20	Top	4 × 3	= 12	Bottom	4 × 3	= 12	Left	3 × 5	= 15	Right	3 × 5	= 15	<p>Students should identify the faces that have equal area.</p> <p style="text-align: center;">Students should then total these.</p> <table style="margin-left: auto; margin-right: auto;"> <tr><td>Front</td><td>5 × 4</td><td>= 20</td></tr> <tr><td>Top</td><td>4 × 3</td><td>= 12</td></tr> <tr><td>Left</td><td>3 × 5</td><td>= 15</td></tr> <tr><td colspan="2" style="text-align: right;">Total</td><td>= 47</td></tr> </table> <p style="text-align: center;">Surface Area = 2 × 47 = 94cm<sup>2</sup></p>	Front	5 × 4	= 20	Top	4 × 3	= 12	Left	3 × 5	= 15	Total		= 47
Front	5 × 4	= 20																															
Back	5 × 4	= 20																															
Top	4 × 3	= 12																															
Bottom	4 × 3	= 12																															
Left	3 × 5	= 15																															
Right	3 × 5	= 15																															
Front	5 × 4	= 20																															
Top	4 × 3	= 12																															
Left	3 × 5	= 15																															
Total		= 47																															

# VOLUME

GENERAL TEACHING & LEARNING POINTS			
Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
Students should use the 3D Geometric Shapes to understand the different faces.	Students should draw the cross section, find the area and multiply by the length.	Students should find the area of the cross section and multiply by the length.	Students should state and use a formula.
Example(s)	Example(s) - These should be seen in books		
A cuboid has a length of 4cm, a width of 2cm and a height of 6cm. Work out the volume of the shape.			
<p>Students should use the 3D Geometric Shapes to understand the different faces and the idea of a cross-section.</p> <p>The Maths Pad interactive tool could be used to help understanding.</p> <p>Students should understand that the volume is the area of the cross-section multiplied by the number of layers.</p> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div>	<div style="text-align: center;">  </div> <p style="text-align: center;">Cross Section</p> $A = bh$ $A = 2 \times 6$ $A = 12\text{cm}^2$ <p style="text-align: center;">Volume =</p> <p style="text-align: center;">Area of Cross-Section X Length</p> $V = 12 \times 4 = 48\text{cm}^3$	<p style="text-align: center;">Volume =</p> <p style="text-align: center;">Area of Cross-Section X Length</p> $\text{Area of CS} = 2 \times 6 = 12\text{cm}^2$ $V = 12 \times 4 = 48\text{cm}^3$	<p style="text-align: center;"><math>V = lwh</math></p> $V = 4 \times 2 \times 6 = 48\text{cm}^3$

# TRANSFORMATIONS

## GENERAL TEACHING & LEARNING POINTS

- Students should understand the idea of a vector and writing and describing vectors prior to performing transformations
- Students should understand the meaning of the terms Object and Image, and this terminology should be used throughout the unit.

## Translations

### Stage 1 Physical

Students should use a physical, cut out, shape to move the object to the image's location.

### Stage 2 Pictorial

Students should draw on their grid how the shape will move. Students should do this from each point.

### Stage 3 Semi-Abstract

Students should draw the vector that is to be applied to each point.

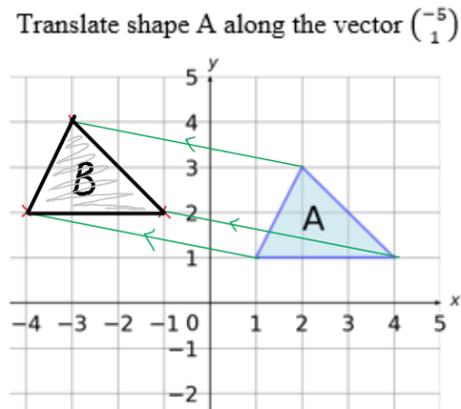
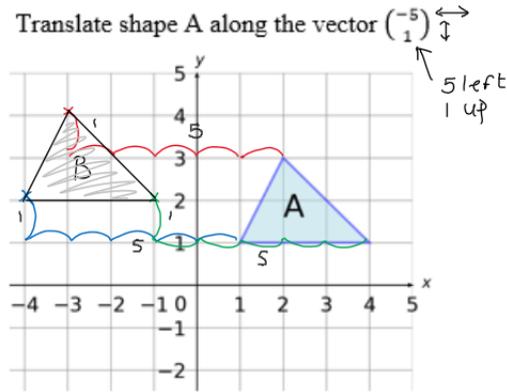
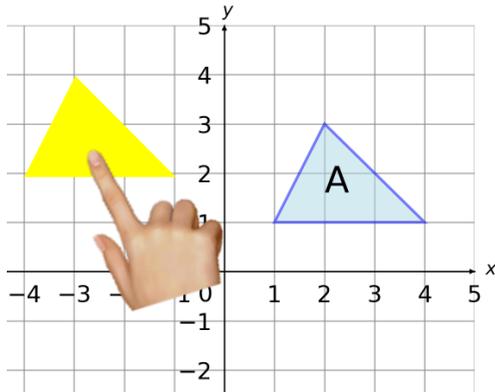
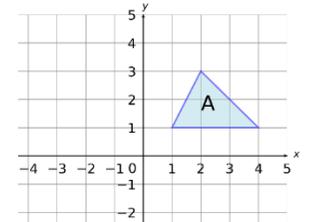
### Stage 4 Abstract

Students should understand that a vector has a fixed origin and therefore the points can be determined through vector arithmetic.

### Example(s)

### Example(s) - These should be seen in books

Translate shape A (shown on the right) along the vector  $\begin{pmatrix} -5 \\ 1 \end{pmatrix}$



$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} -5 \\ 1 \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 4 \\ 1 \end{pmatrix} + \begin{pmatrix} -5 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} -5 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$$

# Reflections

Stage 1  
Physical

Stage 2  
Pictorial

Stage 3  
Semi-Abstract

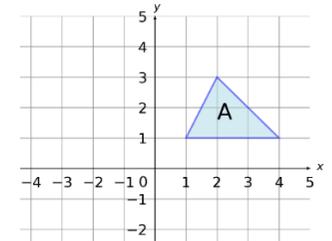
Stage 4  
Abstract

Due to the pictorial nature of reflections, the process outlined below satisfies the Physical, Pictorial and Semi-Abstract stages.

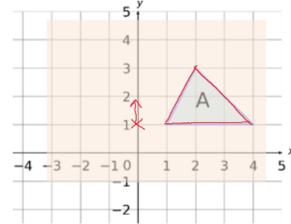
Example(s)

Example(s) - These should be seen in books

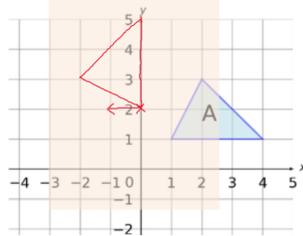
Rotate shape A  $90^\circ$  anti-clockwise about the point (0, 1)



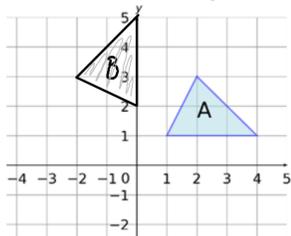
Students should clearly mark the centre of rotation and draw an upward arrow from it.  
Students should trace over the shape, as shown below.



Students should then rotate the tracing paper so that the arrow points in the required direction after the rotation and then trace over the shape.



Students should ensure that their final image is drawn clearly with a ruler.

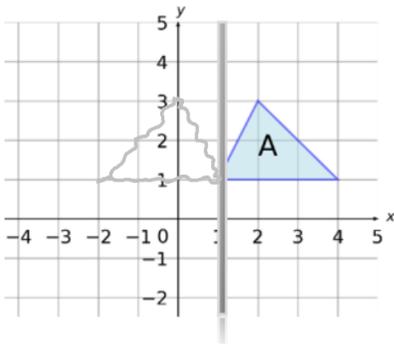
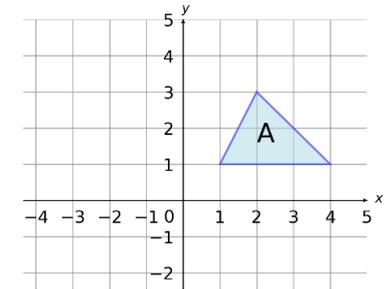


*The full Abstract Stage of rotations requires knowledge of **Matrices** which is not covered at GCSE.*

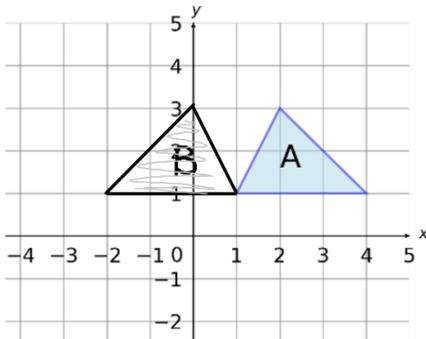
# Reflections

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
Students should use a physical mirror to reflect the shape.	Students should use tracing paper to reflect the shape.	Students should draw the vector that is to be applied to each point.	Students should understand that a vector has a fixed origin and therefore the points can be determined through vector arithmetic.
Example(s)	<b>Example(s) - These should be seen in books</b>		

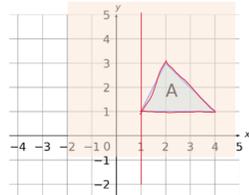
Reflect the shape A in the line  $x = 1$ .



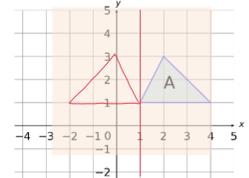
Students should then accurately draw the image with a ruler.



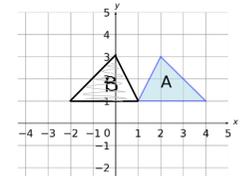
Students should use tracing paper to trace the shape **and** the mirror line.



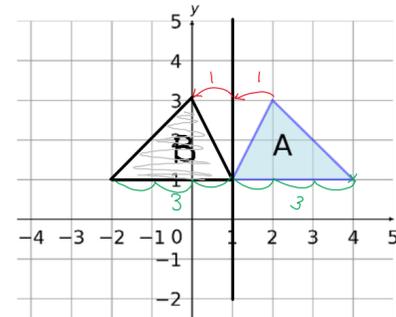
Students should flip the tracing paper over and line up the mirror line.



Students should then accurately draw the image with a ruler.



Students should be able to perform the reflection by determining how to get from each vertex of the shape and then performing the appropriate same step to get to the point on the image.

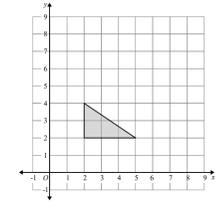


*The full Abstract Stage of Reflections requires knowledge of **Matrices** which is not covered at GCSE.*

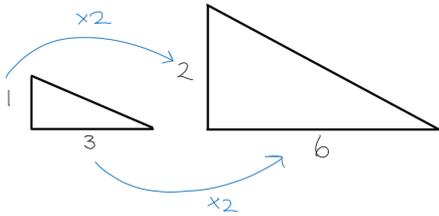
# Enlargements

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
Students should be able to determine the dimensions of a shape after an enlargement.	Students should use a ray diagram to draw the enlargement	Students should determine the vector from the centre of rotation to each vertex, then perform that vector the required number of times.	Students should be able to multiply the vector by the SF of the enlargement.
<b>Example(s)</b>	<b>Example(s) - These should be seen in books</b>		

Enlarge the shape on the right by a scale factor of 2, centred at (1, 0).

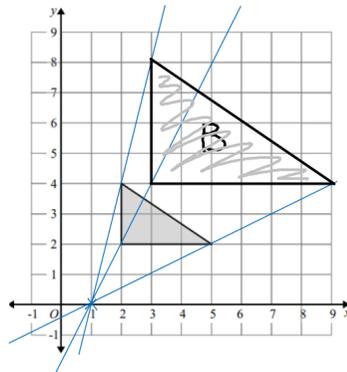


Students should be able to determine and draw the size of the enlarged shape.

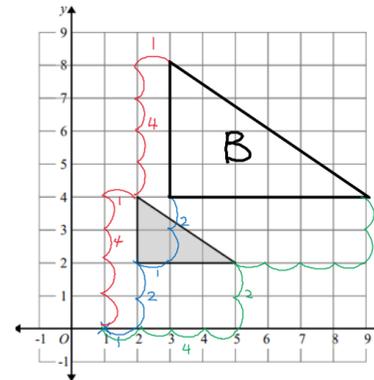


Students should draw straight lines through each vertex of the shape and the centre of enlargement.

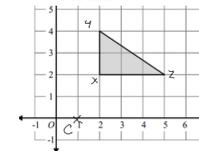
Students should understand what dimensions the enlarged shape will have.



Students should determine the translation from the centre of enlargement to each vertex of the shape and repeat this the same number of times as the scale factor.



Students should label the centre of enlargement C and the vertices of the shape using different letters.

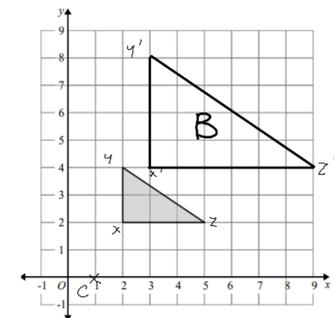


Students should then use scalar vector multiplication to determine the vector from the centre to that point on the image.

$$\vec{CX} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \text{ so } \vec{CX'} = 2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$$\vec{CY} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} \text{ so } \vec{CY'} = 2 \begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 8 \end{pmatrix}$$

$$\vec{CZ} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \text{ so } \vec{CZ'} = 2 \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$$



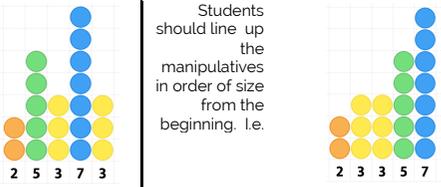
**Curriculum Area:**  
**STATISTICS**

**Mean, Mode, Median and Range**

# MEAN, MODE, MEDIAN AND RANGE

Stage 1 Physical	Stage 2 Pictorial	Stage 3 Semi-Abstract	Stage 4 Abstract
Students should be able to represent a problem using manipulatives (counters or multi-link cubes). It may be useful for students to use different colours for the different values.	Students should draw the manipulatives as a diagram in their books.	Students should be comfortable with the formal definitions of mode, mean, median and range at this point.	Students should be able to find the mean, mode, median and range without needing a diagram. Higher attaining students should link to algebra during working.
Example(s)	Example(s) - These should be seen in books		

Find the mode, range, median and mean of 2, 5, 3, 7, 3.



Students should line up the manipulatives in order of size from the beginning. I.e.

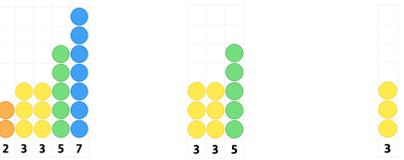
**Mode**  
If different colours have been used this is the colour used for the most columns. If colours aren't used then students should use which height is the most common.

**Range**  
Students should count the difference between the biggest value and the smallest value.

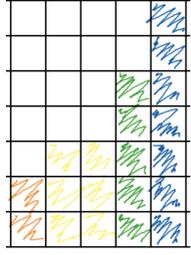
**Mean**  
Students should aim to level out the manipulatives into a rectangle, removing one from the largest to the smallest. The mean is the height of each bar at the end of the process. It is important to highlight that the mean can be a decimal at this point.



**Median**  
Students should remove the smallest and biggest column and repeat. The median is the column that is remaining. Where there are two columns remaining students should find the mean of the two columns as above.

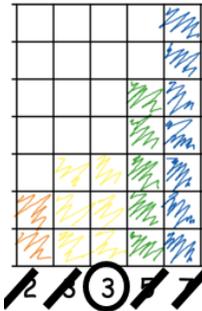


Students should represent the numbers as bars using the squares in their books. This should be smallest to largest.



**Mode, Range and Mean**  
As Physical Stage.

**Median**  
Students should cross off the numbers below the bars smallest then largest. Students should circle the median value.



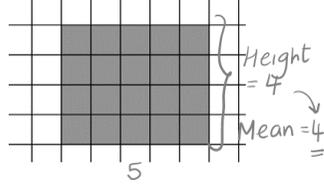
**Mode**  
Students at this point should know the mode is the most common value and should be able to determine this by inspection from the list of numbers.

**Range**  
Students at this point should know the range is **Largest - Smallest**  
Students should clearly show this calculation, i.e.

$$7 - 2 = 5$$

**Mean**  
At this point students should have identified the relationship that the total number of blocks/counters forms the area of the rectangle and the mean is the height of the rectangle.

This should lead to the understanding that the mean is the total number divided by the amount of numbers.



**Median**  
Students at this point should know the median in the middle number.  
Students should be able to find the median without the aid of a diagram.  
Students should cross off the numbers, smallest then largest. Students should circle the median value.



**Mode**  
As semi-abstract stage.

**Range**  
As semi-abstract stage.

**Mean**  
Students should be confident with the definition and calculating the mean as below:

$$\frac{2 + 3 + 3 + 5 + 7}{5}$$

Higher attaining students should be introduced to the formula written using sigma and notation, i.e.

**Median**  
Students at this point should know the median in the middle number.  
Students should be able to state which number is the middle number using the formula

$$\text{Median Number} = \frac{n + 1}{2}$$

In this case:

$$\frac{n + 1}{2} = \frac{5 + 1}{2} = 3$$

So the median is the 3<sup>rd</sup> number in the list when the values are put in order.

